M 382D: Differential Topology
Spring 2015

Exercise Set 11
Due: Mon May 4

Exercises marked with $(\star)$ will definitely not be graded.

Exercise 1. Guillemin/Pollack: Chapter 3, §3 (p. 116): 16 (to understand the hint you need to read pages 113-114), 17, 18, (19 $(\star)$, 20 $(\star)$).

Exercise 2. Guillemin/Pollack: Chapter 3, §4 (p. 130): 2, 5, 10, 11.

Exercise 3. Describe (by formulas or by drawing a picture) a vector field on S^2 which has exactly three zeroes.

Exercise 4. Show that if V is a complex vector space then the underlying real vector space $V_{\mathbb{R}}$ admits a canonical orientation.

(This is the beginning of a very nice story: if M is a complex *manifold* then it also admits a canonical orientation, and this construction is automatically compatible with transverse intersections. In particular all intersection numbers turn out to be *nonnegative* for complex submanifolds intersecting transversally. However, the *self-intersection* number of a complex submanifold can be negative: this occurs when the transversal perturbations are not complex submanifolds anymore.)

Exercise 5. Show that the Lie bracket obeys Jacobi identity: for $X, Y, Z \in \mathfrak{X}(M)$,

$$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0.$$

Exercise 6. $(\star)$ Let $X, Y \in \mathfrak{X}(\mathbb{A}^n)$ compactly supported. Let $\phi_{t,X} : \mathbb{A}^n \to \mathbb{A}^n$ denote the flow of the vector field X for time t. Show that

$$\lim_{t\to 0} \frac{1}{t^2} \left(\phi_{t,Y} \circ \phi_{t,X}(0) - \phi_{t,X} \circ \phi_{t,Y}(0)\right) = [X,Y](0).$$

(Hint: Taylor expand everything around 0.)