<u>Ex</u> $\chi(S^2) = 2$.

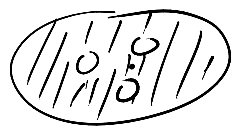
Prop $f: \mathbb{C} \to \mathbb{C}$ hol, $W \subset \mathbb{C}$ region, $f \neq 0$ on ∂W:

$$\#\text{zeros of } f \text{ in } W = \int_{\partial W} \omega \qquad \omega = \frac{f'(z)\,dz}{2\pi i\, f(z)} \qquad dz = dx + i\,dy$$

(ω is complex 1-form)

Pf If f no zero in W then ω extends into W and $d\omega = 0$ [use Cauchy-Riemann] [basis $\partial_z, \partial_{\bar{z}}$]

so $\displaystyle\int_{\partial W} \omega = \int_W d\omega = 0.$

So reduce to case of  an isolated zero z_0, $W = B_\varepsilon(z_0)$.

$f: \partial W \to \mathbb{C}^\times \ni w$

$$\omega = f^*\left(\frac{dw}{2\pi i w}\right)$$

$$\frac{dw}{w} = d\log r + i\,d\theta \qquad w = re^{i\theta}$$

$$\frac{dw}{2\pi i w} = \frac{d\theta}{2\pi} + \text{exact} \qquad \int f^*\left(\frac{dw}{w}\right) = \int f^*\left(\frac{d\theta}{2\pi}\right)$$

$$f(z) = (z-z_0)^k g(z) \qquad g(z_0) \neq 0$$

homotopy $f_t(z) = (z-z_0)^k g_t(z)$ $\quad \begin{cases} g_0 = g \\ g_1(z) = g(z_0) \end{cases}$