

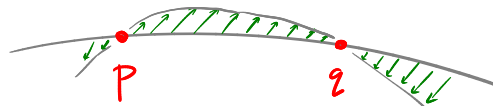
Conjugate points

Def γ geodesic joining p and q :



q is conjugate to p along γ if \exists a Jacobi field J along γ with $J(p) = J(q) = 0$ but $J \neq 0$.

Then, order of conjugacy = $\dim \{ J \text{ Jacobi along } \gamma, J(p) = J(q) = 0 \}$.



Prop If $p \in M, V \in T_p M, q = \exp V$, then

\exp_p is local diffeo in a nbhd of $V \iff q$ is not conjugate to p along $\gamma(t) = \exp(tV)$

Pf For $W \in T_V T_p M \simeq T_p M$,

$$(\exp_p)_*(W) = \frac{d}{ds} \Big|_{s=0} \exp_p(V + sW)$$

$$\text{Set } T'(s,t) = \exp_p t(V + sW)$$

$J(t) = (\partial_s T')(0,t)$ is Jacobi field with $J(1) = \exp_p^*(W)$

So, p and q conjugate along $\gamma \iff (\exp_p)_*$ not injective at V .

