Functions

A function f is a machine which takes an input x and produces an output $f(x)$.

In this course, x and $f(x)$ are always real numbers.

Ex $f(x) = 1 - x^2$

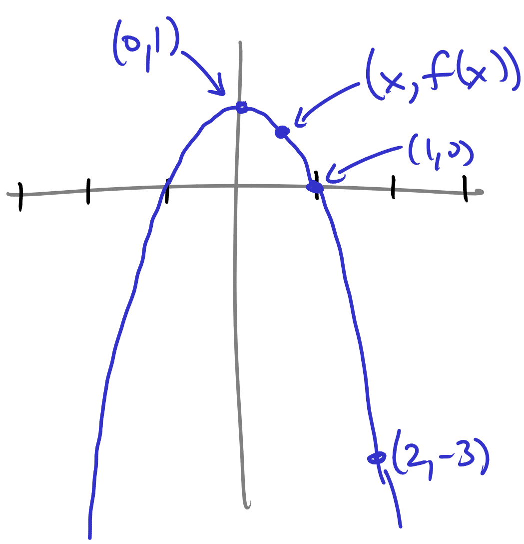
The domain of f is the set of all allowed x — all x such that $f(x)$ exists.

Ex for $f(x) = 1 - x^2$, domain is all real x

$$= \{x : -\infty < x < \infty\}$$
$$= (-\infty, \infty)$$

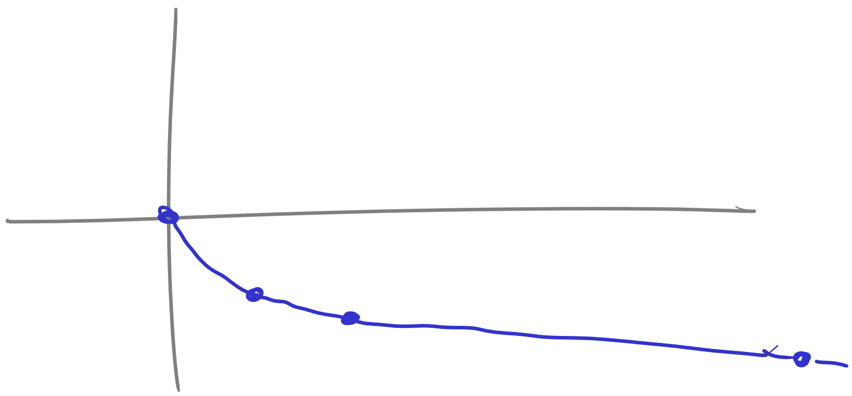
The range of f is the set of all y such that $y = f(x)$ for some x.

Ex $f(x) = 1 - x^2$: range is $(-\infty, 1] = \{y : -\infty < y \leq 1\}$



x	$f(x)$
0	1
1	0
2	-3
3	-8
-1	0
-2	-3
⋮	

Ex $f(x) = -\sqrt{x}$: domain $= \{x : x \geq 0\} = [0, \infty)$



x	$f(x)$
0	0
1	-1
2	$-\sqrt{2}$
16	-4

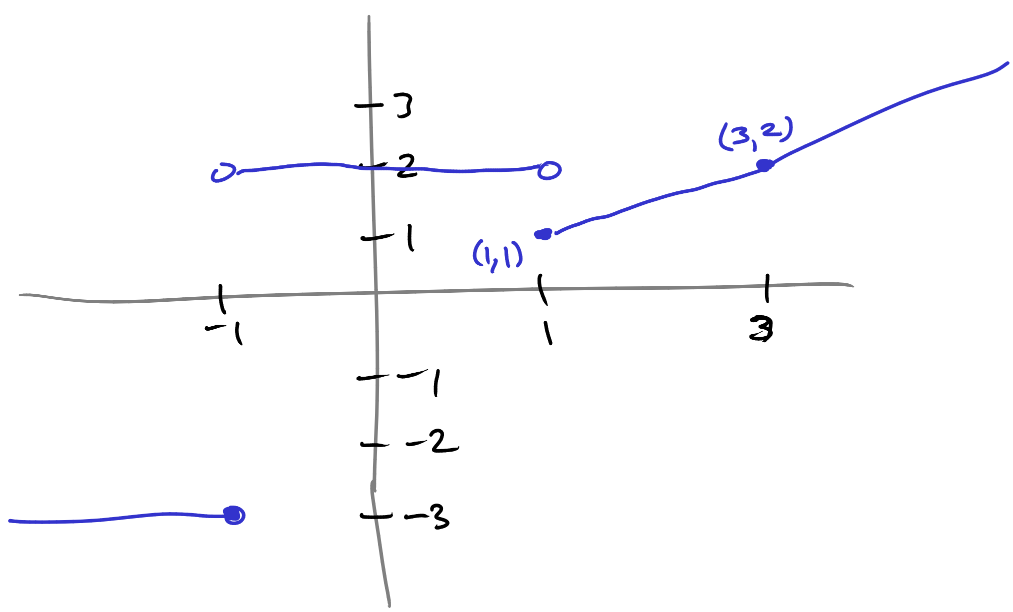
range $= (-\infty, 0]$

Ex

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & \text{if } x \geq 1 \\ 2 & \text{if } -1 < x < 1 \\ -3 & \text{if } x \leq -1 \end{cases}$$

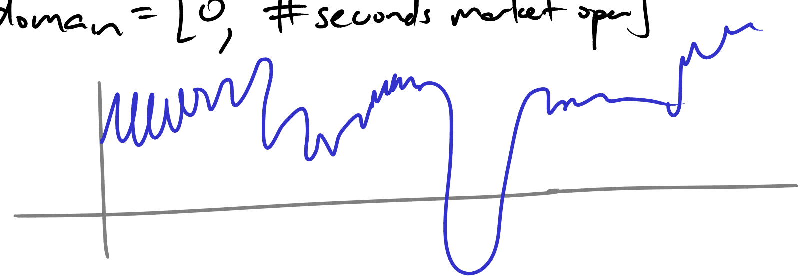
domain $= (-\infty, \infty)$

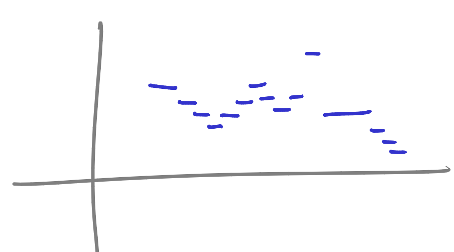
range $= \{-3\} \cup [1, \infty)$



Ex $f(x) =$ value of DJIA today, x seconds past opening

domain $= [0, \#\text{seconds market open}]$

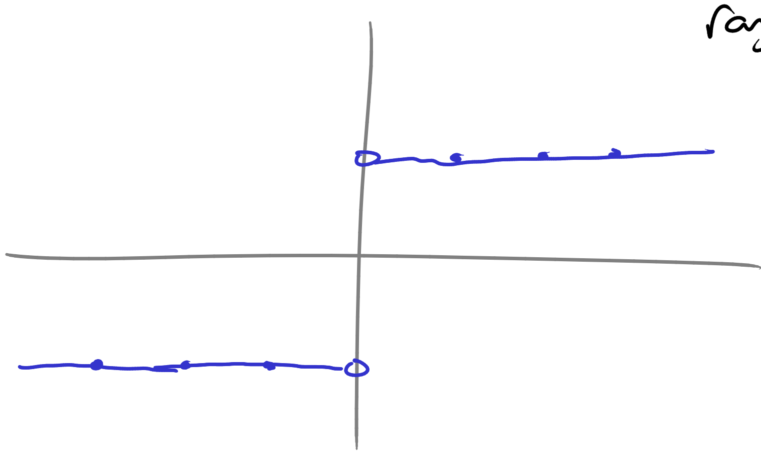




Ex $f(x) = \dfrac{|x|}{x}$

domain $= (-\infty, 0) \cup (0, \infty)$
$= \{x : x \neq 0\}$

range $= \{-1\} \cup \{1\}$



x	$f(x)$
1	$\frac{\lvert 1\rvert}{1} = \frac{1}{1} = 1$
2	$\frac{\lvert 2\rvert}{2} = \frac{2}{2} = 1$
3	$\frac{\lvert 3\rvert}{3} = \frac{3}{3} = 1$
-1	$\frac{\lvert -1\rvert}{-1} = \frac{1}{-1} = -1$
-2	$\frac{\lvert -2\rvert}{-2} = \frac{2}{-2} = -1$
	$\vdots$

if $x > 0$, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$

if $x < 0$, $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$

So $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

Similarly $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Classes of functions

Linear: $f(x) = ax + b$ ex $f(x) = 7x + 4$

Polynomial: $f(x) = ax^n + bx^{n-1} + \cdots + z$ ex $f(x) = -2x^4 + 7x^2 + 8x - 10$

Rational: $f(x) = \dfrac{P(x)}{Q(x)}$ P, Q polynomials   ex $f(x) = \dfrac{x^9 - 7x^7 + 1}{\frac{7}{3}x^{1000} - 8.3\pi}$

Power: $f(x) = x^a$

ex $f(x) = x^2$

$f(x) = x^{1/2} = \sqrt{x}$

$f(x) = x^{-3} = \frac{1}{x^3}$

$f(x) = x^{2/5} = \sqrt[5]{x^2} = \left(\sqrt[5]{x}\right)^2$

Trig functions: $f(x) = \sin(x)$

$\cos(x)$

$\tan(x)$

$\vdots$

Exponential function: $f(x) = a^x$ $\quad a = \text{constant},\ a > 0$

Ex $f(x) = 2^x$

$f(x) = \pi^x$

What does a^x really mean?

If $x = \frac{p}{q}$ p, q integers we know what a^x means: $a^x = a^{p/q}$

$= \sqrt[q]{a^p}$

What if x is not rational?

e.g. what is 2^π? $\quad \pi \approx 3.1415926\ldots$

$$2^3 < 2^{3.1} < 2^{3.14} < 2^{3.141} < 2^{3.1415} < \cdots < 2^\pi$$

$$2^4 > 2^{3.2} > 2^{3.15} > 2^{3.142} > 2^{3.1416} > \cdots > 2^\pi$$

2^π can be defined as the unique real # obeying these inequalities!

<u>Fact</u> By this procedure, we can define a^x for <u>any</u> real x.

<u>Laws of exponents</u>

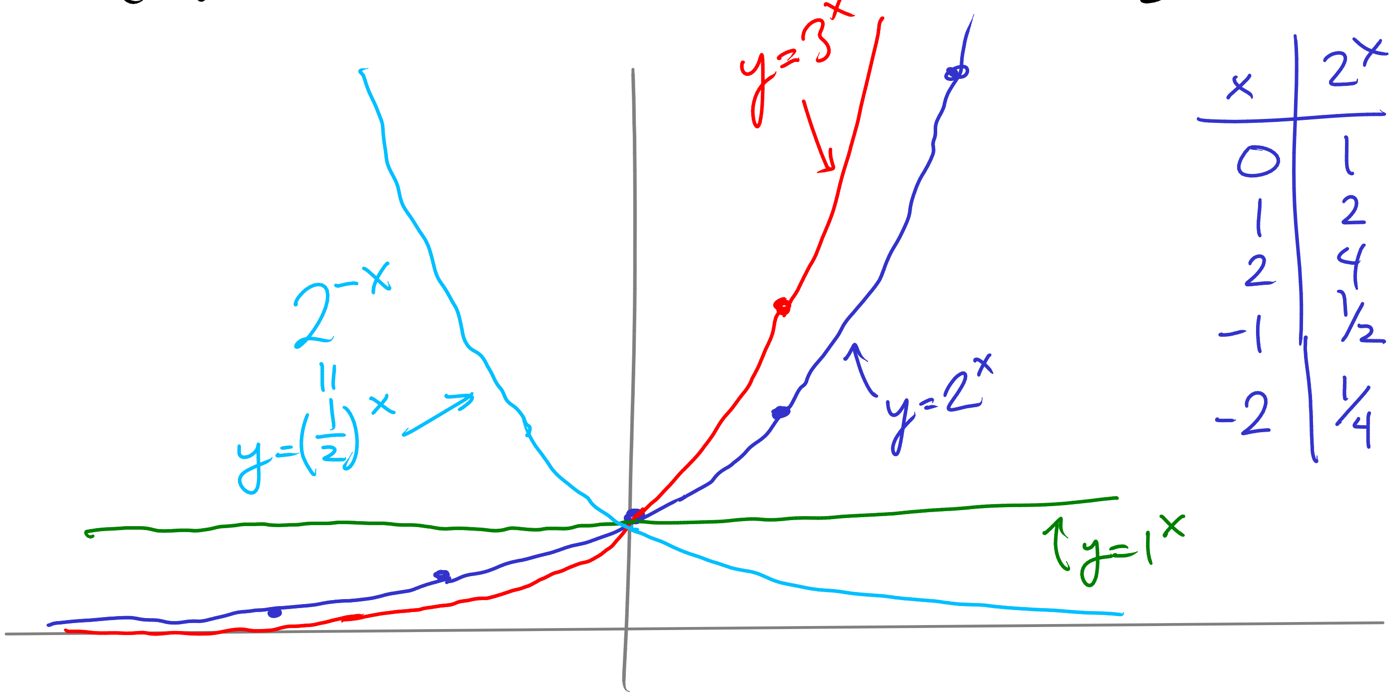
1. $a^{x+y} = a^x a^y$
2. $a^{-x} = \frac{1}{a^x}$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$

<u>Ex</u>

$$\frac{1}{a^{9/4}} = a^{-9/4}$$

$$\frac{1}{x^{-7}} = x^7$$

$$9 \cdot 3^x = 3^2 \cdot 3^x = 3^{2+x}$$



x	2^x
0	1
1	2
2	4
-1	1/2
-2	1/4

For <u>any</u> function $f(x)$, the graph of $y=f(x)$

$y=f(-x)$

are related by <u>reflection</u> in the y-axis.

Special case: if $f(x) = f(-x)$ call f even

if $f(-x) = -f(x)$ call f odd

Ex $\quad f(x) = \dfrac{1+x^2}{x^6}$: $\quad f(-x) = \dfrac{1+(-x)^2}{(-x)^6} = \dfrac{1+x^2}{x^6} = f(x)$

so f is even

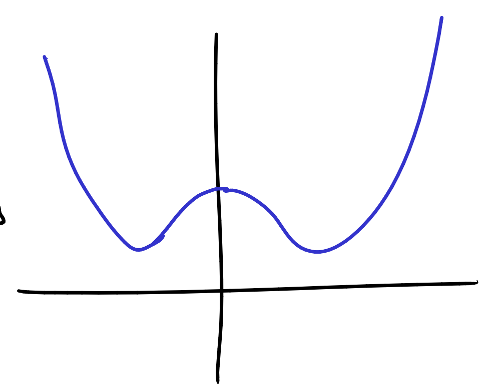
$f(x) = \dfrac{x}{1+x^2} \quad f(-x) = \dfrac{-x}{1+(-x)^2} = \dfrac{-x}{1+x^2} = -f(x)$

so f is odd

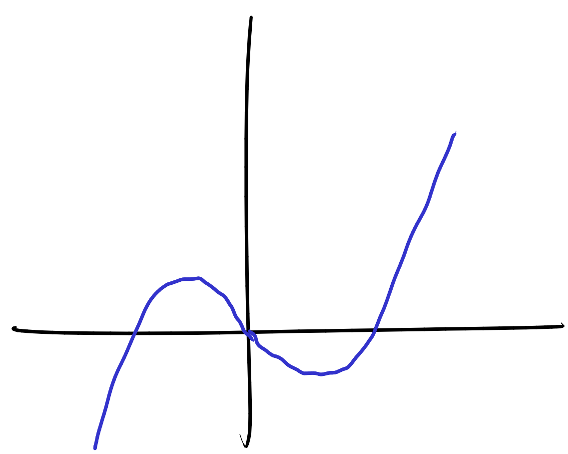
$f(x) = x + x^2$ neither even or odd

$f(-x) = -x + x^2$

Even functions



Odd function



Fact If $a \neq 1$ and $a^x = a^y$
then $x = y$.

Ex If $5^x = 25^{3x-2}$

then $5^x = (5^2)^{3x-2}$

$$5^x = 5^{6x-4}$$

$$x = 6x - 4$$

$$-5x = -4$$

$$\underline{\underline{x = 4/5}}$$