$x^3 + x^2y + 4y^2 = 6$

$1 + x = \sin(xy^2)$

Find y' using implicit differentiation

- Implicit Differentiation
- Derivative of inverse functions
- Derivative of log functions

1. $\frac{d}{dx}(x^3 + x^2y + 4y^2) = \frac{d}{dx}(6)$

$3x^2 + x^2y' + 2xy + 8yy' = 0$

$y'(x^2 + 8y) = -3x^2 - 2xy$

$y' = -\dfrac{3x^2 + 2xy}{x^2 + 8y}$

2. $\frac{d}{dx}(1 + x)\ \frac{d}{dx}(\sin(xy^2))$ find y'

$1 = \cos(xy^2) + [y^2 + 2yy'x]$

$\dfrac{1}{\cos(xy^2)} = y^2 + 2yy'x$ $\quad \dfrac{1}{\cos(xy^2)} - y^2 = 2yy'x$

distribute $\quad y' = \dfrac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$

$$\dfrac{\dfrac{1}{\cos(xy^2)} - y^2}{2yx} = y'$$

3. $3x^2 + 2y^2 = 5$

$6x + 4yy' = 0$

$-6x = 4yy'$

$\dfrac{-3x}{2y} = y' = \dfrac{-3}{2} \cdot \dfrac{x}{y}$ $\quad y'' = \dfrac{-3}{2}\ \dfrac{y - y'x}{y^2}$

plug in y'

$$\boxed{\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}}$$

Derivative of inverse trig functions

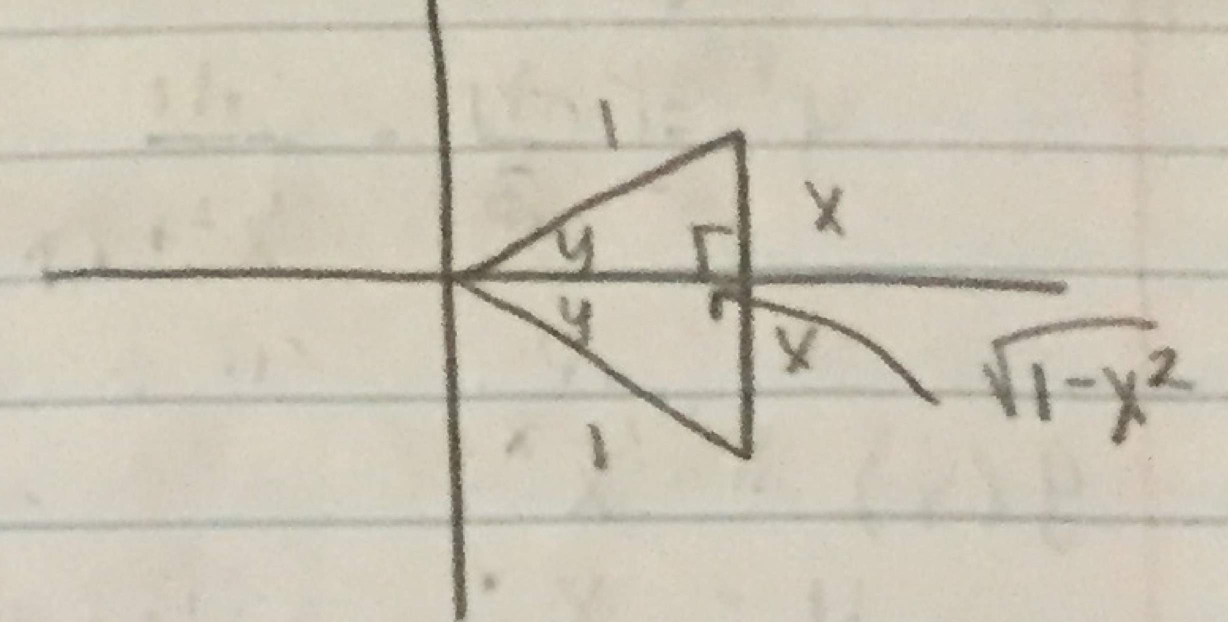
$y = \sin^{-1}x$ find dy/dx $\qquad -\pi/2 \le y \le \pi/2$

$\sin y = x$

$\cos y \cdot y' = 1$

$y' = \frac{1}{\cos y}$

$d/dx \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$



$y = \tan^{-1}x$ $\qquad 1 + \tan^2\theta = \sec^2\theta$

$\tan y = x$

$\sec^2 y \cdot y' = 1$

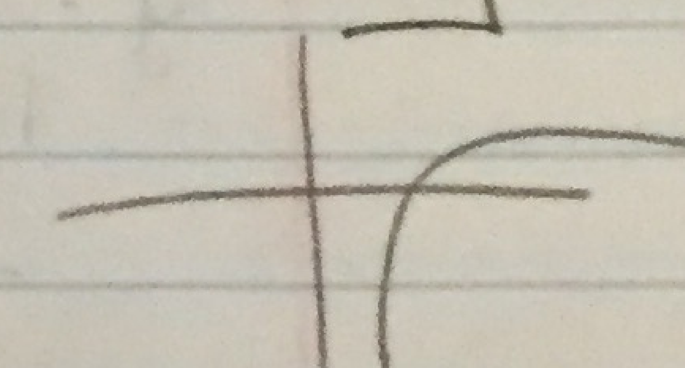
$y' = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

$$\boxed{\frac{d}{dx}a^x = a^x \ln a}$$

Derivative of log functions

$y = \ln x$ Domain: $(0,\infty)$

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$



$e^y = x$

$e^y \cdot y' = x$

$y' = \frac{x}{e^y} = \frac{1}{x}$

$y = \log_a x$

$a^y = x$

$a^y \ln a \cdot y' = 1$

$$\boxed{\frac{d}{dx}\log_a x = \frac{1}{x \ln a}}$$

$y' = \frac{1}{a^y \ln a} = \frac{1}{x \ln a}$

ex) $g(x) = \frac{7^x}{x}$ $\qquad f: 7^x \quad f': 7^x \ln 7 \qquad g: x \quad g': 1$ $\qquad y = \frac{\ln x}{x^2}$

$\frac{7^x \ln 7 (x) - 7^x}{x^2}$

$y = \frac{\ln x}{x^2}$

$y' : \frac{1 - 2\ln x}{x^3}$ $\qquad y'' : \frac{6\ln x - 5}{x^4}$

$y = \ln \sqrt[3]{x^2+7x}$

$y = \frac{1}{3} \ln (x^2+7x)$

$y' = \frac{1}{3} \cdot \frac{1}{x^2+7x} \cdot (2x+7)$

$\left[\begin{array}{l} \ln ab = \ln a + \ln b \\ \ln a^b = b \ln a \end{array} \right]$

$g(x) = x^x$

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{d}{dx} \quad \frac{1}{y} y' = 1 + \ln x$

$y' = y(1+\ln x) \qquad y' = x^x(1+\ln x)$

$y = (\ln x)^{\cos x}$

$\ln y = \cos x \cdot \ln(\ln x)$

$\frac{1}{y} y' = \cos x \frac{1}{\ln x} \cdot \frac{1}{x} - \sin x \ln(\ln x)$

$y' = (\ln x)^{\ln x} \left[\frac{\cos x}{x \ln x} - \sin x \ln(\ln x) \right]$

$y = \ln |x| \quad \text{Dom } x \neq 0$

$\ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$

$\frac{d}{dx} \ln|x| = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{1}{x} & x < 0 \end{cases}$