Lecture 2 | 31 Aug 2018

Last time: functions $f(x)$

$$\underset{\text{real number}}{x} \longrightarrow \boxed{f} \longrightarrow \underset{\text{real number}}{f(x)}$$

A function f is called even if for all x in the domain,

$$f(-x) = f(x)$$

f is called odd if for all x in the domain,

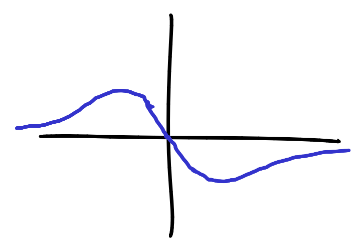
$$f(-x) = -f(x)$$

Ex $f(x) = \dfrac{2+x^2}{x^4+x^6}$ $\qquad f(-x) = \dfrac{2+(-x)^2}{(-x)^4+(-x)^6} = \dfrac{2+x^2}{x^4+x^6} = f(x)$

$\longrightarrow$ f is even

$f(x) = \dfrac{-x}{1+x^2}$ $\qquad f(-x) = \dfrac{-(-x)}{1+(-x)^2} = \dfrac{x}{1+x^2} = -f(x)$

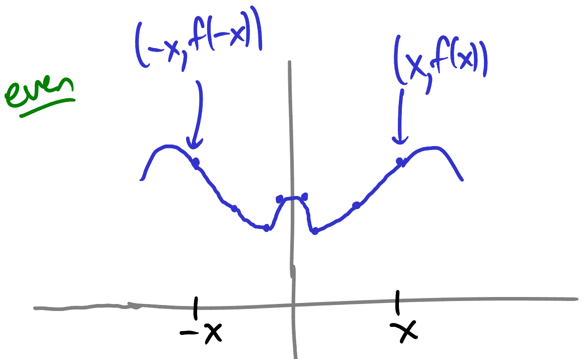
$\longrightarrow$ f is odd

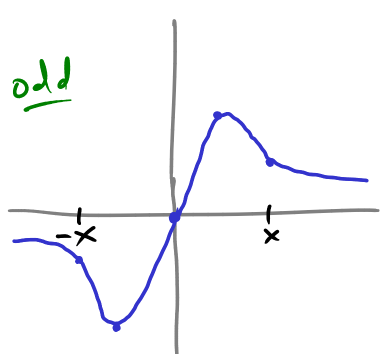


$f(x) = x + x^2$ $\qquad f(-x) = (-x) + (-x)^2$

$= -x + x^2$

$\longrightarrow$ f not even or odd





for odd:

$f(x) = -f(-x)$

$f(0) = -f(-0)$

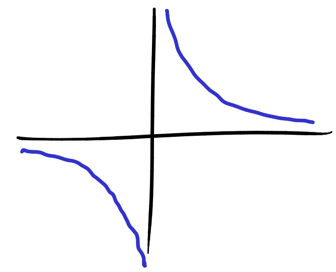
$\Rightarrow 2f(0) = 0$

$f(0) = 0$ ✓

so if 0 is in domain of odd f
then $f(0)=0$

but 0 might not be in domain

eg $f(x)=\frac{1}{x}$



Classes of functions

Linear $f(x) = ax+b$ ex $f(x)=7x-4$

Polynomial $f(x) = ax^n+bx^{n-1}+\cdots+z$ ex $f(x)= -2x^4+6x^3+9x-\frac{17}{2}$

Rational $f(x)=\frac{P(x)}{Q(x)}$ P,Q polynomial   ex $f(x)=\frac{3+7x-x^4}{8\pi-x^{1000}}$

Power $f(x)=x^a$ ex $f(x)=x^2$

$f(x)=x^{1/2}=\sqrt{x}$

$f(x)=x^{2/5}=\sqrt[5]{x^2}=\left(\sqrt[5]{x}\right)^2$

Ex $\left(\frac{\sqrt{2}}{2}\right)^x=\left(\frac{2^{\frac{1}{2}}}{2}\right)^x=\left(2^{\frac{1}{2}}\cdot 2^{-1}\right)^x=\left(2^{-\frac{1}{2}}\right)^x=2^{-\frac{x}{2}}$

Exponential $f(x)=a^x$ $a=$ constant, $a>0$

ex $f(x)=2^x$

$f(x)=\pi^x$

What does a^x mean?

If $x=\frac{p}{q}$ p,q integers then $a^x=a^{\frac{p}{q}}=(a^p)^{\frac{1}{q}}=\sqrt[q]{a^p}$

But what if e.g. $x=\pi$? What is 2^π?

Fact: the definition of a^x can be extended to x = any real number!

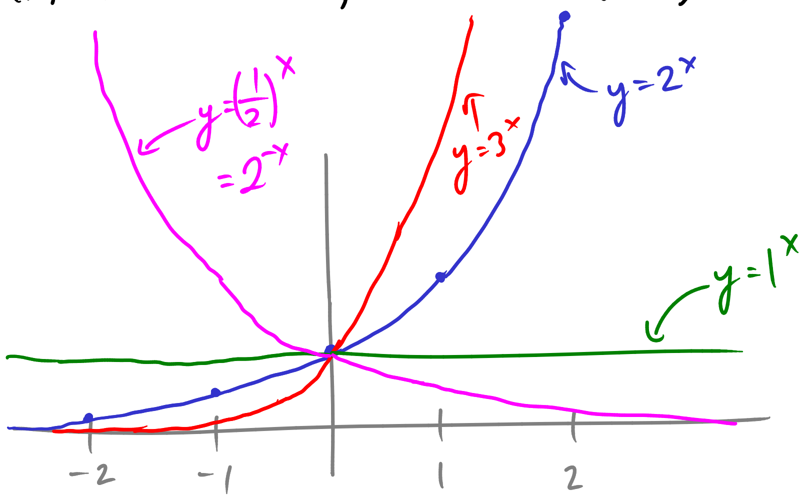
e.g. 2^π makes sense.

$$2^3 < 2^{3.1} < 2^{3.14} < 2^{3.14159} < \dots < 2^\pi$$

$$2^4 > 2^{3.2} > 2^{3.15} > 2^{3.14160} > \dots > 2^\pi$$

2^π is the unique # obeying these inequalities!

So $f(x) = a^x$ makes sense, has domain $(-\infty, \infty)$



x	2^x
0	1
1	2
2	4
3	8
−1	$\frac{1}{2}$
−2	$\frac{1}{4}$
⋮	⋮

NB graph of $y = \left(\frac{1}{2}\right)^x = 2^{-x}$ is obtained by reflecting graph of $y = 2^x$ in y-axis. Similar for any $f(-x) \longleftrightarrow f(x)$

Laws of exponents

① $a^{x+y} = a^x a^y$

② $a^{-x} = \dfrac{1}{a^x}$

③ $(a^x)^y = a^{xy}$

④ $(ab)^x = a^x b^x$

Ex $\dfrac{1}{a^3} = a^{-3}$

$9 \cdot 3^x = 3^2 \cdot 3^x = 3^{x+2}$

Fact If $a > 0$, $a \neq 1$ and $a^x = a^y$ then $x = y$ (the function $f(x) = a^x$ is "1-1")

Ex if $5^x = 25^{3x-2}$ what is x?

$5^x = (5^2)^{3x-2}$

$$5^x = 5^{6x-4}$$
$$x = 6x-4$$
$$-5x = -4$$
$$\underline{\underline{x = \tfrac{4}{5}}}$$

Composition of functions

If f, g are functions, with $\text{range}(g) \subset \text{domain}(f)$,
then we can define a new function $f \circ g$
by the formula $(f \circ g)(x) = f(g(x))$

Ex $f(x) = x^2 + 1$
$g(x) = 6x$

then $(f \circ g)(x) = f(g(x))$
$= f(6x)$
$= (6x)^2 + 1 = 36x^2 + 1$

Ex $f(x) = \sin x$
$g(x) = |x|$

$(f \circ g)(x) = f(g(x))$
$= f(|x|)$
$= \sin |x|$
$= \begin{cases} \sin x & x > 0 \\ \sin(-x) = -\sin x & x < 0 \end{cases}$

$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$