Lecture 21 19 Oct 2018

Our final exam set: Dec 17, 9–12 (in this room)

Next week I'm called for jury duty Monday: Prof. Bob Gompf

Mean Value Theorem

Suppose someone drives from Austin to San Antonio (80 mi)

in Austin at 1:30pm

in San Antonio at 2:00pm

Then they must have been speeding at some time between 1:30 and 2:00

because their average speed was $\frac{80 \text{ mi}}{0.5 \text{ hr}} = 160 \text{ mph}$

Fact: Suppose f is a function continuous on $[a,b]$, differentiable on (a,b).

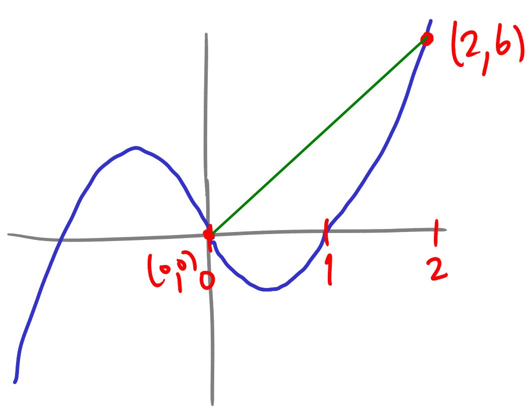
Then there is some c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$f(t)$ = position of car at time t

ie, there is some c in (a,b) where the slope of the tangent line to graph $y=f(x)$ at $(c, f(c))$ equals slope of the secant line connecting $(a, f(a))$ to $(b, f(b))$!

Ex $f(x) = x^3 - x$



slope of secant line $= \frac{6-0}{2-0} = 3$

MVT sez: there is some c in $(0,2)$ such that $f'(c) = 3$

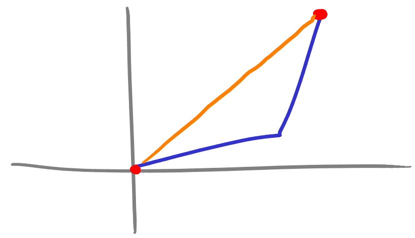
Let's check: $f'(x) = 3x^2 - 1$

$f'(c) = 3 \Rightarrow 3c^2 - 1 = 3$

$3c^2 = 4$

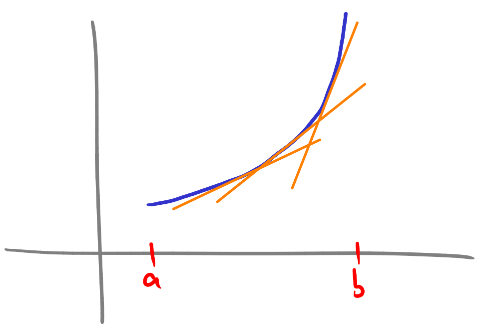
$c^2 = \frac{4}{3} \qquad c = \pm\frac{2}{\sqrt{3}} \qquad c = \frac{2}{\sqrt{3}} \in (0,2)$ ✓

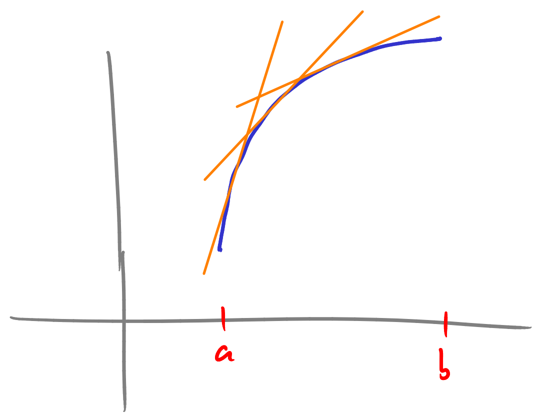
It's important that f is diff'ble in MVT!



<u>Graphing using derivatives, cont'd</u>

<u>Concavity</u>

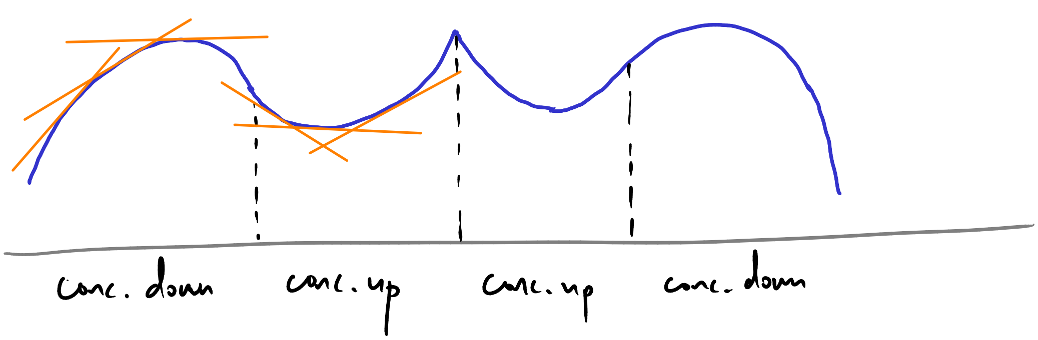




Both of them have $f'(x) > 0$ for $x \in (a,b)$ but they are different:

Graph of $y = f(x)$ is <u>concave up</u> if it lies <u>above</u> all its tangent lines in (a,b)

——— <u>concave down</u> ——— <u>below</u> ——— (a,b).



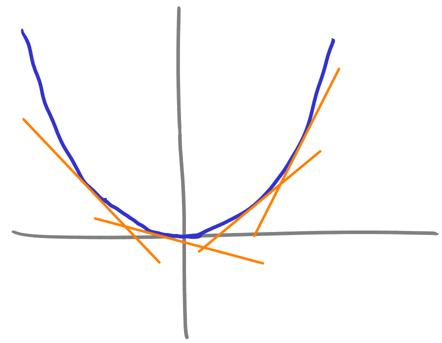
<u>Fact</u> If $f''(x) < 0$ for all x in (a,b) then graph of $y = f(x)$ is <u>conc down</u> on (a,b)

— $f''(x) > 0$ ——— <u>conc up</u> ———.

Ex $f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$ so $f''(x) > 0$ for all $x \in (-\infty, \infty)$



So graph of $y = x^2$ is concave up for all x in $(-\infty, \infty)$.

[Why? if $f''(x) > 0$ then slope of tangent line increases as x increases]

A point of inflection is a point $(c, f(c))$ where f is continuous and the graph $y = f(x)$ changes from conc. up to conc. down or vice verse.

Ex $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

	f incr		f decr		f increasing
$f'(x)$:	$+$		$-$		$+$
x		-1		1	

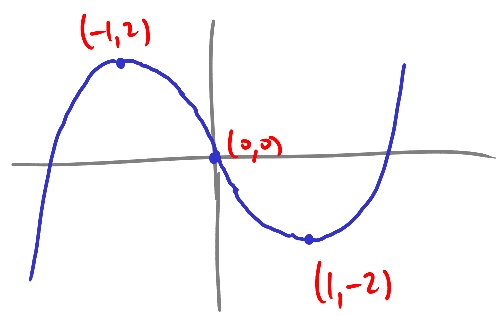
$f''(x) = 6x$

	f conc down		f concave up
$f''(x)$:	$-$		$+$
x		0	

So: $x = 1$ loc min $f(1) = -2$

$x = 0$ inflection pt $f(0) = 0$

$x = -1$ loc max $f(-1) = 2$



Second Derivative Test

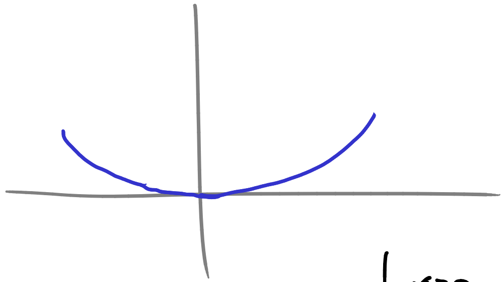
If f is twice diff'ble at c, $f'(c) = 0$, and

① $f''(c) > 0$, then c is local min

② $f''(c) < 0$, ——— local max

③ $f''(c) = 0$, then the test fails — no info

Q Find $f(x)$ s.t. for some c, $f'(c) = 0$, $f''(c) = 0$ but c is a local min!



$$y = x^4 = f(x)$$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2$$

at $c=0$,

$f(c)=0$ $\quad f'(c)=0$ $\quad f''(c)=0$ $\quad f'''(c)=0$ $\quad f^{(4)}(c)=24$