Lecture 24

26 Oct 2018

Midterm 2 next Fri Nov 2

Format: like Midterm 1

Covers Lectures 12–24
ie sections 3.4–4.7 (optimization)
(chain rule)

My office hrs next week:
M 2–3
W 3:30–4:30 RLM 9.134
extra → Th 10:30–11:30

Summary of Curve-Sketching

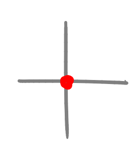
To sketch the graph $y = f(x)$ for some given $f(x)$, what do we look at?

(A) domain
(B) x- and y-intercepts
(C) symmetry? (is f even, odd, periodic?)
(D) asymptotes
i) horizontal
ii) vertical
(E) where increasing / decreasing?
(F) local min/max?
(G) concavity / points of inflection?
(H) sketch

Ex $f(x) = \dfrac{x^3}{x^3+1}$

(A) domain: all x except where $x^3+1=0$, ie all x except $x=-1$
$(-\infty,-1) \cup (-1,\infty)$

(B) intercepts: y-intercept $f(0) = \frac{0}{0+1} = 0$ $(0,0)$
x-intercepts at $f(x)=0$, ie $x^3=0$ ie $x=0$ $(0,0)$



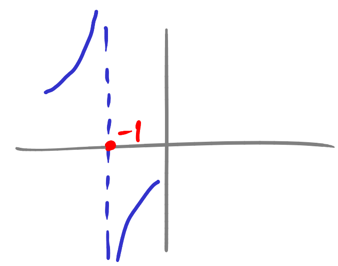
(C) symmetry: is it even/odd? $f(-x) = \dfrac{(-x)^3}{(-x)^3+1} = \dfrac{-x^3}{-x^3+1} \neq f(x)$ or $-f(x)$
$\Rightarrow$ not even or odd

Ⓓ asymptotes: i) could have vertical asymp at $x=-1$

$$\lim_{x\to -1^+} \frac{x^3}{x^3+1} = ?$$ say $x = -1 + (\text{small}_{+ve})$ eg -0.99

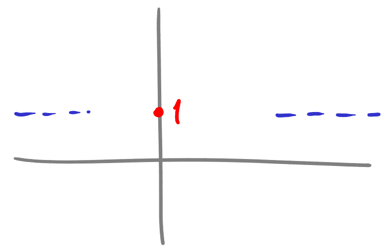
then $\frac{x^3}{x^3+1} \approx \frac{-1}{\text{small} +ve} = \text{big } -ve$

so $$\lim_{x\to -1^+} \frac{x^3}{x^3+1} = \underline{-\infty}$$



Similarly $$\lim_{x\to -1^-} \frac{x^3}{x^3+1} = \cdots = \underline{+\infty}$$

ii) horiz asymptotes? $$\lim_{x\to +\infty} \frac{x^3}{x^3+1} = 1$$
$$\lim_{x\to -\infty} \frac{x^3}{x^3+1} = 1$$



Ⓔ where increasing/decreasing?

$$f(x) = \frac{x^3}{x^3+1} \qquad f'(x) = \cdots = \frac{3x^2}{(x^3+1)^2}$$

	incr		incr		incr
f'	+		+		+
x		-1		0	

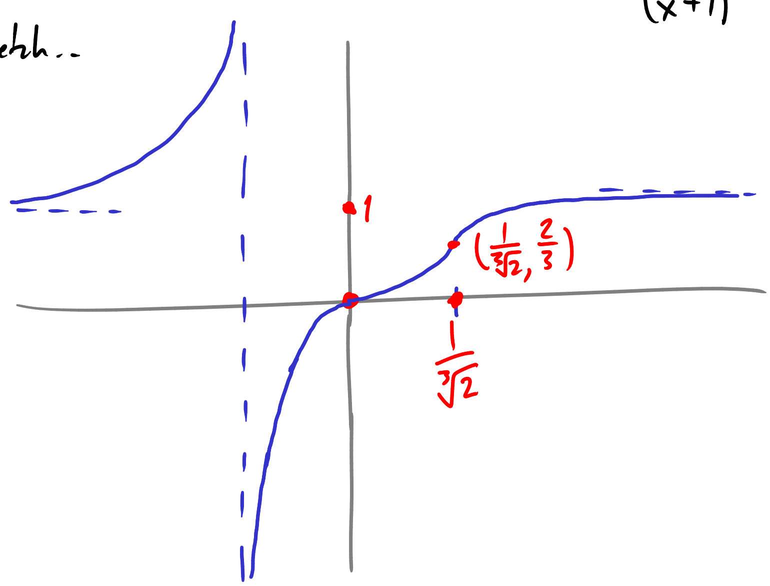
Ⓕ local min/max? $\underline{\underline{\text{none}}}$

Ⓖ concavity/inflection? $$f''(x) = \cdots = \frac{6x - 12x^4}{(x^3+1)^3} = \frac{6x(1-2x^3)}{(x^3+1)^3}$$

$\rightarrow$ $\underline{\text{infl pts}}$ at $x = 0, \frac{1}{\sqrt[3]{2}}$

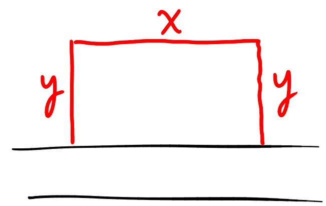
	conc ↑		conc ↓		conc ↑		conc ↓
f''	+		−		+		−
x		-1		0		$\frac{1}{\sqrt[3]{2}}$	

Ⓗ Sketch..



Optimization

A school is built on a lot adjacent to a straight creek. The school wants to build a rectangular playground using 800 ft of fencing.



What's the biggest area that can be enclosed by the fence?

area (in ft^2)
length of horizontal side (in ft)

Need to find absolute max of the function $A(x)$

What's the domain? $0 < x < 800$

$A(x) = xy$ and $x + 2y = 800$ so $y = \frac{800-x}{2}$

so $A(x) = x \cdot \frac{800-x}{2}$

$= 400x - \frac{1}{2}x^2$

To find possible absolute max, check
1) places where $A'(x)$ DNE
2) ——— $A'(x) = 0$
3) endpoints

1) none

2) $A'(x) = 400 - x$ so $A'(x) = 0$ means $400 - x = 0$ ie $x = 400$

$A(400) = 400 \cdot 400 - \frac{1}{2}(400^2)$

$= \frac{1}{2}(400^2) = 80000$ sq ft

3) endpoints: $A(0) = 0$

$A(800) = 0$

So, the optimal area is 80000 sq ft, attained at $x = 400$ ft
$y = 200$ ft