Lecture 26

Oct 31 2018

Midterm 2 covers Lectures 12-24
but no "optimization" pbs

Last time: Newton's Method

To find a solution of $f(x)=0$

start with an initial approximate solution x_1

then iteratively "improve":

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\vdots$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

<u>Ex</u> Find $\sqrt[6]{2}$ to 4 decimal places.

$f(x) = x^6 - 2$

$f'(x) = 6x^5$

Think of this as finding solution to $\underbrace{x^6 - 2}_{f(x)} = 0$

Take initial approximation: $x_1 = 1$

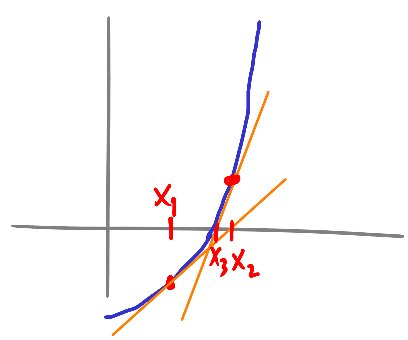
then $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{-1}{6} = \frac{7}{6} \approx 1.16667$

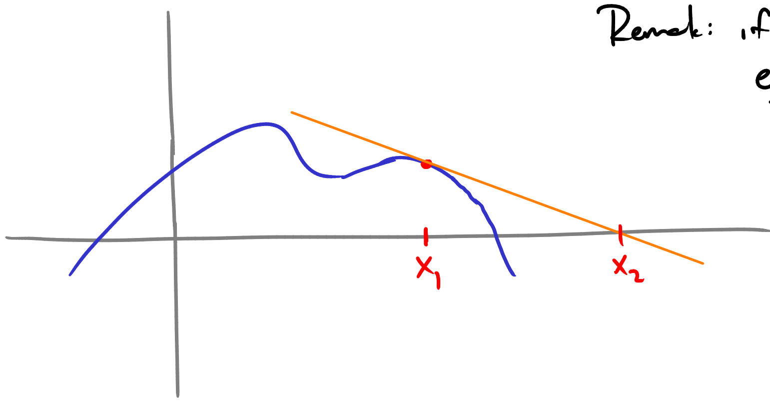
$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \cdots \approx 1.1264$

$x_4 = \cdots \approx 1.1225$

$x_5 = \cdots \approx 1.1225$

then stop: our approx. sol^n is <u>1.1225</u>





Remark: if the initial x_1 is not close enough to the solution, Newton's method may not work!

Antiderivatives

- $f(x) = x^2 \rightsquigarrow f'(x) = 2x$
- $f(x) = \sin(x^3) \rightsquigarrow f'(x) = 3x^2\cos(x^3)$

Suppose we want to "go backward".

$$f \underset{\text{antiderivative}}{\overset{\text{derivative}}{\rightleftarrows}} f'$$

We say $F(x)$ is an antiderivative for $f(x)$ if $F'(x) = f(x)$.

Ex $f(x) = x$ has antiderivative $\frac{1}{2}x^2$

or $\frac{1}{2}x^2 + 6$

or $\frac{1}{2}x^2 + 999$

$\vdots$

To get all possible antiderivatives of $f(x)$, first find one antideriv, then add an arbitrary constant (usually called C)

Ex $f(x) = \cos x$ has general antiderivative $F(x) = \sin x + C$

$f(x) = x^n$ " " " $F(x) = \dfrac{x^{n+1}}{n+1} + C$ ← if $n \neq -1$

$$\left(\frac{d}{dx} F(x) = \frac{1}{n+1} \cdot (n+1)\, x^n = x^n = f(x)\right)$$

$f(x) = \dfrac{1}{x}$ " " " $F(x) = \ln|x| + C$

$f(x) = \frac{1}{1+x^2}$ '   "   " $F(x) = \tan^{-1} x + C$

Build more complicated ex. from these:

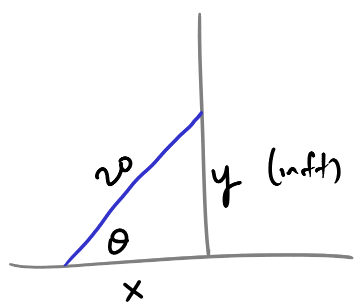
$$f(x) = 9x^2 + 6x^{3/2} - \frac{2}{x^4} + \cos 2x$$

has gen'l antideriv:

$$F(x) = 3x^3 + 6\left(\frac{1}{5/2} x^{5/2}\right) - 2\left(\frac{1}{(-3)} x^{-3}\right) + \frac{1}{2} \sin 2x + C$$

Next time: uses of these

<u>Related rates</u>



20-ft ladder sliding down wall

<u>height</u> decreasing at 1 ft/s

when the distance <u>from</u> the wall is 3 ft

how fast is the angle between ladder and ground changing?

<u>know</u>: $\frac{dy}{dt} = -1$

$x = 3$

<u>want to know</u>: $\frac{d\theta}{dt}$

$$\sin\theta = \frac{y}{20}$$

$$\frac{d}{dt}(\sin\theta) = \frac{d}{dt}\left(\frac{y}{20}\right)$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{20} \cdot \frac{dy}{dt}$$

and $\cos\theta = \frac{x}{20}$

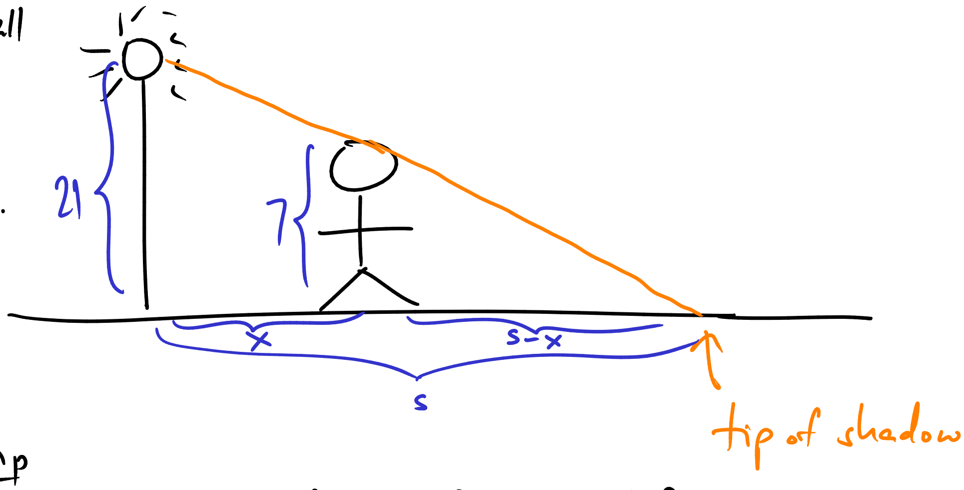
$$\frac{x}{20} \cdot \frac{d\theta}{dt} = \frac{1}{20} \frac{dy}{dt}$$

$$x \frac{d\theta}{dt} = \frac{dy}{dt}$$

<u>now plug in</u>: $3 \cdot \frac{d\theta}{dt} = -1$ so $\underline{\underline{\frac{d\theta}{dt} = -\frac{1}{3}}}$

Frankenstein ← 7 ft tall walks directly away from a light at 2 ft/s.

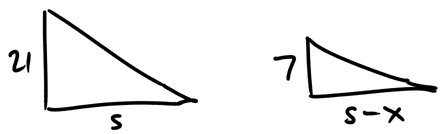
The light is on a 21-ft pole.



How fast is the tip of his shadow moving when he is 10 ft away from the pole?

Know: $\frac{dx}{dt} = 2$ $\quad x = 10$ $\qquad$ Want: $\frac{ds}{dt}$

Similar triangles:



$$\frac{s}{21} = \frac{s-x}{7}$$

$$s = 3(s-x)$$

$$s = 3s - 3x$$

$$-2s = -3x$$

$$s = \frac{3}{2}x$$

$$\frac{ds}{dt} = \frac{3}{2}\frac{dx}{dt} = \frac{3}{2}(2) = \underline{\underline{3}} \text{ ft/s}$$