Lecture 27 — 5 Nov 2018

HW10 due 3am Nov 9 (Friday) ← extension

HW11 will be 3am Nov 13 (Tue) as usual

Midterm 2 grades posted today or tomorrow

Class average ≈ 86% ← good!

Last time: antiderivative

$$\begin{tikzcd} f(x) \arrow[r, bend left, "\text{derivative}"] & f'(x) \arrow[l, bend left, "\text{antiderivative}"] \end{tikzcd}$$

Ex What is the function $F(x)$ which has $F'(x) = 4x+7$

and $F(1) = 6$?

Since $F'(x) = 4x+7$

we have $F(x) = 2x^2 + 7x + C$ for some C

and $F(1) = 6$, so

$$2(1^2) + 7(1) + C = 6$$

$$9 + C = 6$$

$$C = -3$$

thus $\underline{F(x) = 2x^2 + 7x - 3}$

Why care about antiderivatives?

Standard reason:

$$\begin{tikzcd} \text{position } s(t) \arrow[r, bend left, "\text{derivative}"] & \text{velocity } v(t) \arrow[l, bend left, "\text{antiderivative}"] \arrow[r, bend left, "\text{derivative}"] & \text{acceleration } a(t) \arrow[l, bend left, "\text{antiderivative}"] \end{tikzcd}$$

Ex A train accelerates with constant acceleration, $a(t) = 4 \text{ ft}/s^2$

At time $t=0$ it has velocity 100 ft/s.

How far does it go in 20 s?

know: $a(t) = 4$

$v(0) = 100$

$s(0) = 0$

want: $s(20)$.

$s(t)$ is antideriv of $v(t)$

$v(t)$ is antideriv of $a(t)$

$\hookrightarrow v(t) = 4t + C$

and $v(0) = 100$, so $4(0) + C = 100$

$C = 100$

so $v(t) = 4t + 100$

then $s(t) = 2t^2 + 100t + D$

and $s(0) = 0$, so $2(0)^2 + 100(0) + D = 0$

$D = 0$

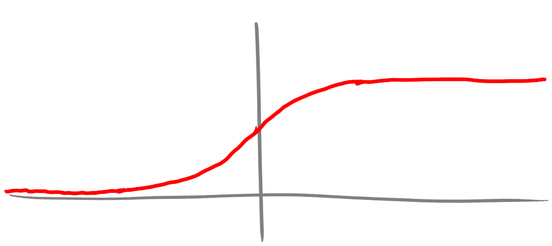
so $s(t) = 2t^2 + 100t$

$s(20) = 2(20)^2 + 100(20) = 800 + 2000 = \underline{2800 \text{ ft}}$

Remark Every continuous function has an antiderivative.

But, e.g. the antiderivative of $f(x) = e^{-x^2}$

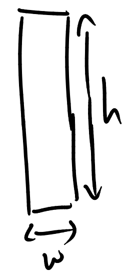
cannot be written in terms of "elementary" functions ($+, \times, -, \div$, exp, log, ln, sin, cos, $\sin^{-1}$, $\cos^{-1}$, ...)



We give this antideriv. a name. "error function" $f(x) = \text{erf } x$

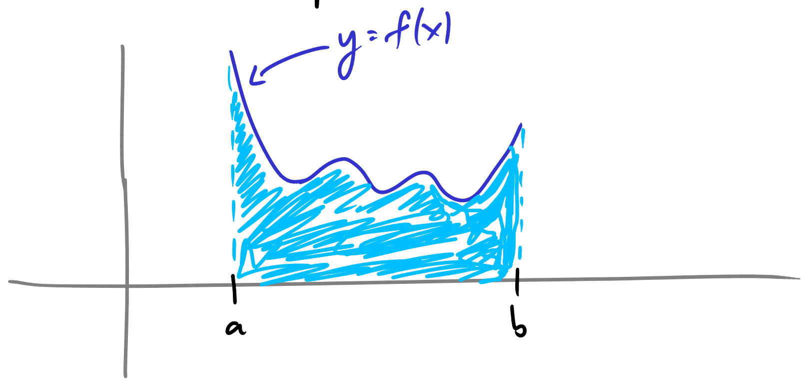
Areas

We all know areas of simple shapes



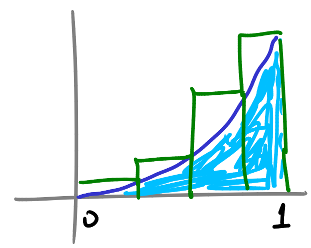
rectangle
$A = w \cdot h$

How about more complicated shapes?

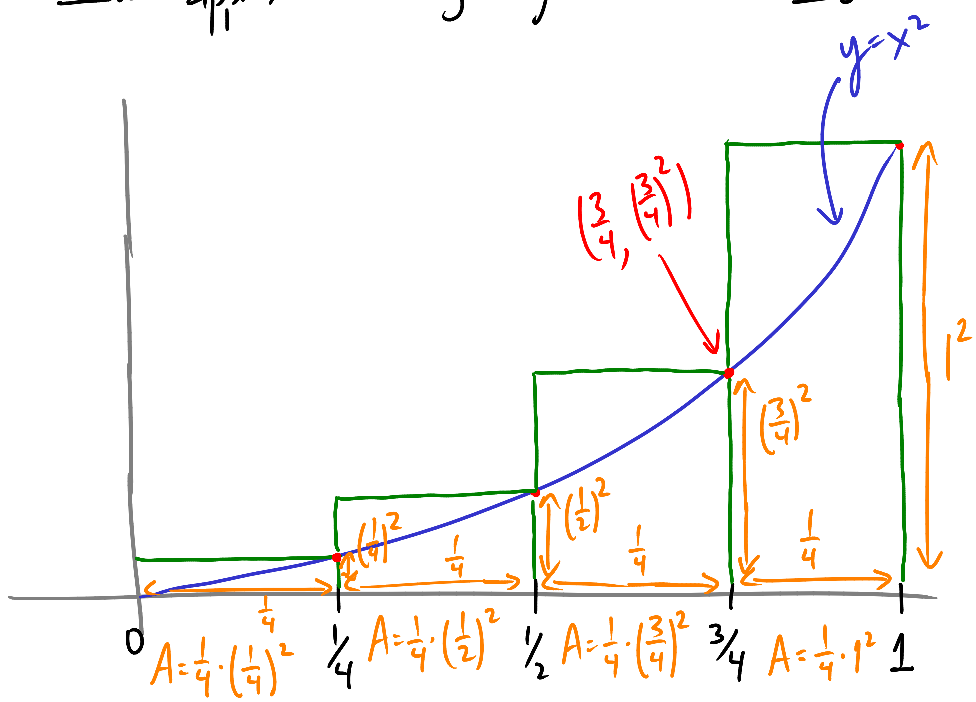


Let's try to calculate the area under the graph of $y=f(x)$, over the x-axis, between $x=a$ and $x=b$.

Ex Say $f(x)=x^2$. Estimate the area of the region between $y=f(x)$ and the x-axis and between $x=0$ and $x=1$.



Idea: approximate our region by a bunch of rectangles.



$$\text{total area of rectangles} = \frac{1}{4}\left[\left(\frac{1}{4}\right)^2+\left(\frac{2}{4}\right)^2+\left(\frac{3}{4}\right)^2+1^2\right]$$

$$= \frac{15}{32}$$

This gives an <u>overestimate</u> of the area under $y = x^2$ from 0 to 1.

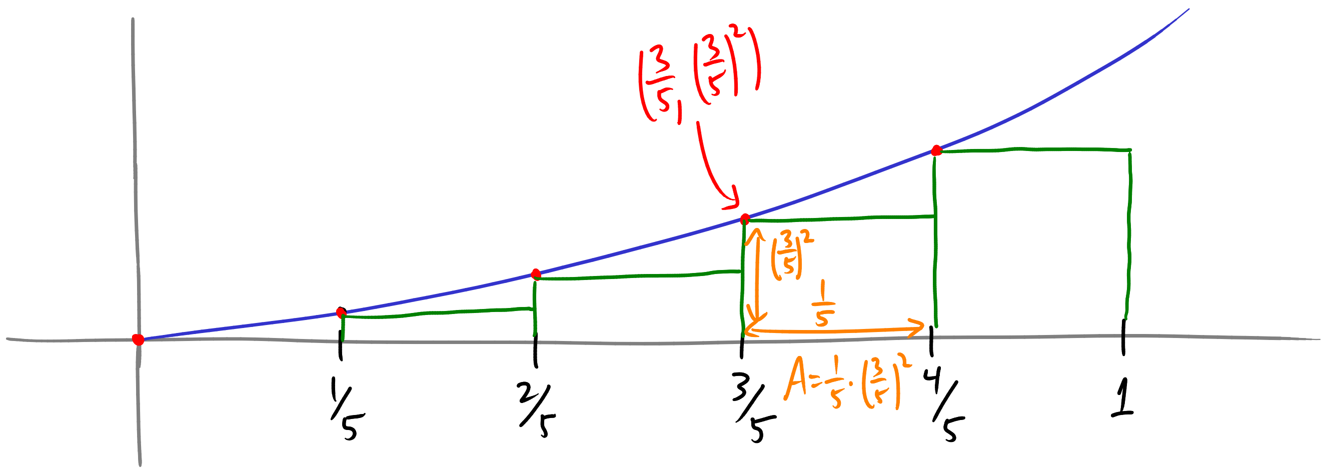
(b/c the rectangles <u>completely cover</u> that area)

It is the "estimated area using <u>4 rectangles</u> and using the <u>right</u> endpoints of the intervals as sample points"

So, call it R_4.

Then $R_4 = \frac{15}{32}$.

<u>Ex</u> Estimate the same area, using 5 rectangles and <u>left</u> endpoints as sample pts.



Estimated area $\quad L_5 = \frac{1}{5}\left(0^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2\right) = \frac{30}{125}$

This is an <u>underestimate</u> of the actual area.