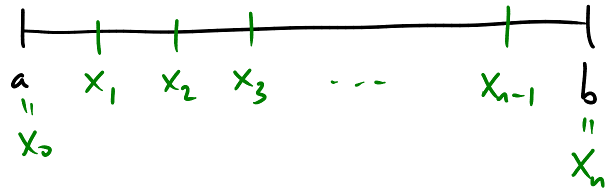
Last time: definition of definite integral

Given a function $f$ on $[a,b]$

divide $[a,b]$ into $n$ equal parts, width $\Delta x = \frac{b-a}{n}$

$$x_i = x_0 + i\Delta x = a + i\,\frac{b-a}{n}$$




Pick "samplepoint" $x_i^*$ in $[x_{i-1}, x_i]$

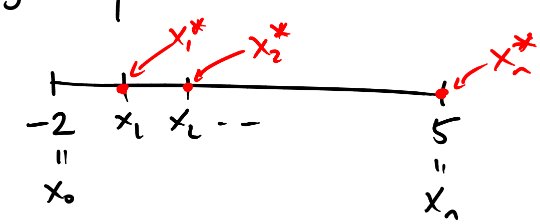
e.g. left endpt. $x_i^* = x_{i-1}$

right endpt. $x_i^* = x_i$

midpt. $x_i^* = x_{i-1} + \frac{1}{2}\Delta x$

Then Riemann sum of $f$ is $\sum_{i=1}^{n} f(x_i^*)\cdot\Delta x$ and $\int_a^b f(x)\,dx = \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(x_i^*)\Delta x\right)$

Ex Write the definition of $\int_{-2}^{5} e^{2x+1}\,dx$ as a limit, using right endpoints.




$a = -2$
$b = 5$

$$\Delta x = \frac{b-a}{n} = \frac{7}{n}$$

$$x_i^* = x_i = x_0 + i\Delta x = -2 + i\frac{7}{n}$$

$$f(x) = e^{2x+1}$$

Riemann sum.

$$\sum_{i=1}^{n} f(x_i^*)\cdot\Delta x = \sum_{i=1}^{n} f\left(-2+\frac{7i}{n}\right)\cdot\frac{7}{n}$$

$$= \sum_{i=1}^{n} e^{2(-2+\frac{7i}{n})+1}\cdot\frac{7}{n}$$

$$= \frac{7}{n}\sum_{i=1}^{n} e^{-3+\frac{14i}{n}}$$

So, $\int_{-2}^{5} e^{2x+1}\,dx = \lim_{n\to\infty} \frac{7}{n}\sum_{i=1}^{n} e^{-3+\frac{14i}{n}}$

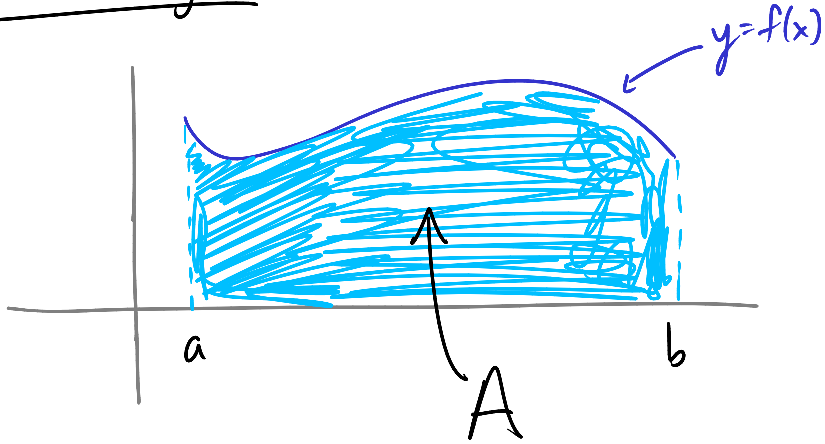
(If we used left endpts, we'd get

$$\int_{-2}^{5} e^{2x+1}\,dx = \lim_{n\to\infty} \frac{7}{n}\sum_{i=1}^{n} e^{-3+\frac{14(i-1)}{n}}$$

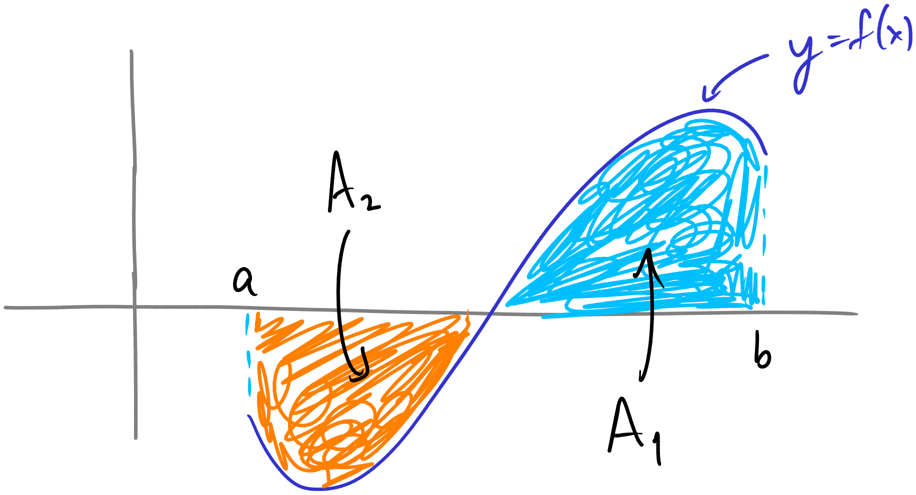
Both are OK!)

---

Facts about integrals



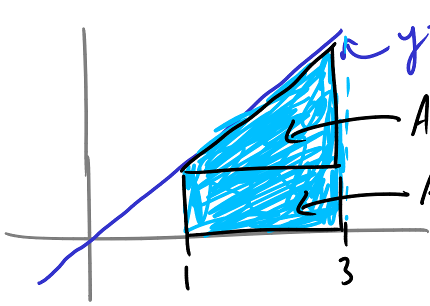

$$\int_a^b f(x)\,dx = A$$




$$\int_a^b f(x)\,dx = A_1 - A_2$$

$$\int_a^b |f(x)|\,dx = A_1 + A_2$$

Ex Evaluate $\int_1^3 x\,dx$ (by interpreting it as an area.)




$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$A = 2\cdot 1 = 2$

$\Big\} A = 2+2 = 4$

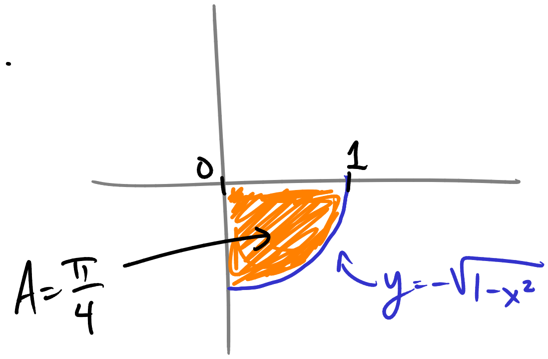
so $\int_1^3 x\,dx = \underline{\underline{4}}$.

Ex Evaluate $\int_0^1 -\sqrt{1-x^2}\,dx$ by interpreting it as an area.

$\left( y=-\sqrt{1-x^2} \quad y^2=1-x^2 \quad y^2+x^2=1 \quad \text{(unit circle)} \right)$

So $\int_0^1 -\sqrt{1-x^2}\,dx = -\frac{\pi}{4}$

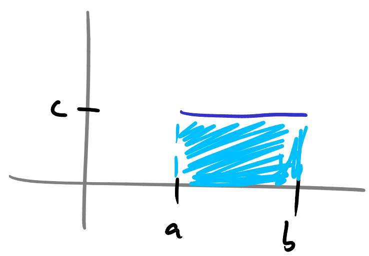
– sign because $f<0$!




---

Basic laws for integrals

1) $\int_a^b c\,dx = c\cdot(b-a)$.

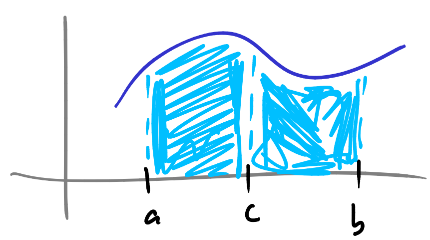



2) $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3) $\int_a^b c\cdot f(x)\,dx = c\cdot\int_a^b f(x)\,dx$

4) $\int_a^b f(x)-g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$ $\quad$ $\left[ 2), 3) \Rightarrow 4) \right]$

5) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$




So far, always took $\int_a^b$ where $b>a$.

Convenient to define $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$.

$\left[ \text{eg we already saw } \int_1^3 x\,dx = 4. \quad \text{So, } \int_3^1 x\,dx = -4. \right]$

---

Q $\int_1^2 f(x)\,dx + \int_1^3 g(x)\,dx = ?$

can "simplify": $= \int_1^2 f(x)\,dx + \int_1^2 g(x)\,dx + \int_2^3 g(x)\,dx$

$$= \int_1^2 \left(f(x)+g(x)\right) dx + \int_2^3 g(x)\,dx$$

---

Ex if $\int_1^3 f(x)\,dx = 4$ and $\int_3^7 f(x)\,dx = 16$

what is $\int_1^7 3f(x)\,dx$?

$$\int_1^7 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^7 f(x)\,dx$$

$$= \quad 4 \quad + \quad 16 \quad = 20$$

$$\int_1^7 3f(x)\,dx = 3\cdot\int_1^7 f(x)\,dx = 3\cdot 20 = \underline{60}.$$

---

Next time: FTC