# Lecture 32

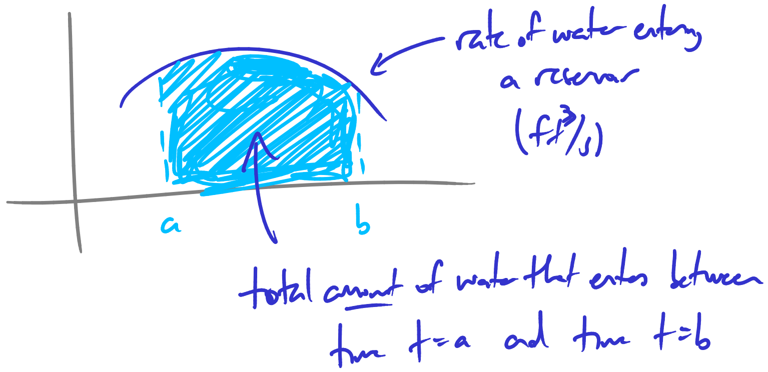
16 Nov 2018

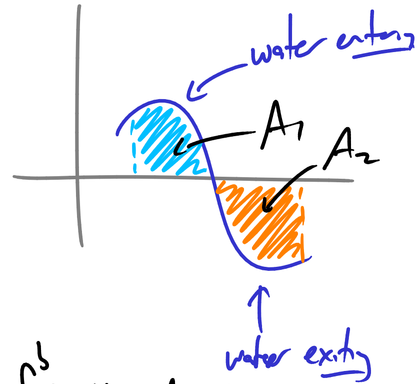
Last time: net change theorem

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

$\int_a^b F'(t)\,dt$: rate of change of F

$F(b) - F(a)$: total change of F



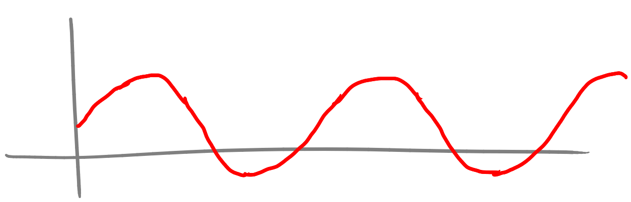




$$\int_a^b f\,dt = A_1 - A_2 = \text{net change in amount of water}$$

Ex A capacitor is connected to a load that can charge or discharge it.
The current flowing into the capacitor is $\sin(\pi t) + \frac{1}{2}$.

ie $Q'(t) = \sin(\pi t) + \frac{1}{2}$ $\quad$ ($Q(t)$ = charge in the cap. at time $t$)

If the cap. starts with 10 units of charge at time $t=0$ $\quad$ ($Q(0) = 10$)
how much does it have at time $t = 6$?



$$Q(6) - Q(0) = \int_0^6 Q'(t)\,dt$$

$$= \int_0^6 \left(\sin(\pi t) + \tfrac{1}{2}\right) dt$$

$$= -\frac{1}{\pi}\cos(\pi t) + \frac{t}{2} \Big|_0^6$$

$$= \left(-\frac{1}{\pi}\cos(6\pi) + 3\right) - \left(-\frac{1}{\pi}\cos(0) + 0\right)$$

$$= \left(-\frac{1}{\pi} + 3\right) - \left(-\frac{1}{\pi}\right)$$
$$= 3.$$

so $\quad Q(6) = 3 + Q(0) = 3 + 10 = \underline{\underline{13}}.$

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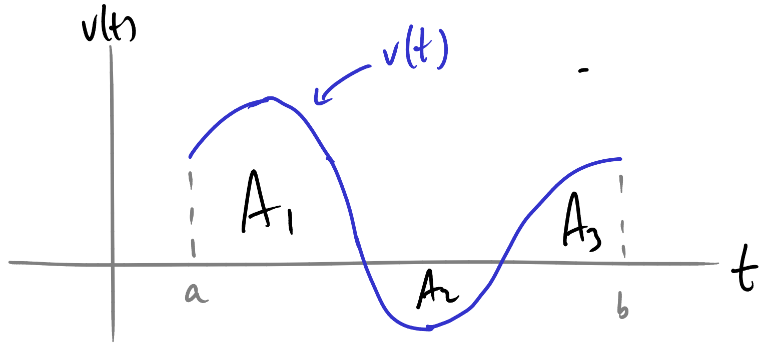
## Total displacement

If $s(t)$ = position along a line
$\quad s'(t) = v(t)$ = velocity

$\left(\begin{array}{ll} v(t) > 0: & \text{moving to the right} \\ v(t) < 0. & \text{moving to the left} \end{array}\right)$




Total displacement $\quad s(b) - s(a) = \int_a^b v(t)\,dt = A_1 - A_2 + A_3$

Total distance
(odometer reading) $\quad \int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$

Ex A particle moves along a line with $v(t) = t^2 - t - 6$ m/s $\quad$ ($t$ in s)
from time $t=1$ to $t=4$.

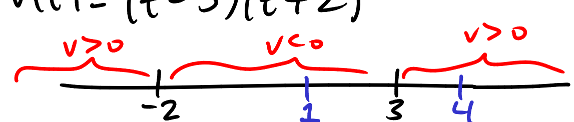
a) What is the total displacement?

$$\Delta s = s(4) - s(1) = \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6\,dt$$
$$= \cdots = -\frac{9}{2}$$
(ie $\frac{9}{2}$ m in the negative dir, ie the left)

b) What is the total distance the particle covers?

$$\int_1^4 |v(t)|\,dt$$

$v(t) = (t-3)(t+2)$



so $\int_1^4 |v(t)|\,dt = \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$

$$= \int_1^3 -t^2+t+6\,dt + \int_3^4 t^2-t-6\,dt$$

$$= \quad \dots$$

$$= 22/3 + 17/6 = \frac{61}{6}\text{ m}$$

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## Method of substitution ("u-substitution")

A way of finding antiderivatives.

Ex $\int \sqrt{2x-3}\,dx = ?$

Try to relate this to sth easier to understand: introduce $u = 2x-3$

Replace x by u everywhere.

$\int \sqrt{2x-3}\,dx = \int \sqrt{u}\,dx$

To relate dx to du:

$\frac{du}{dx} = 2$ so $du = 2\,dx$

so $\frac{1}{2}du = dx$

$$= \int \sqrt{u}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\int \sqrt{u}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{1}{3}u^{3/2} + C \qquad = \underline{\underline{\frac{1}{3}(2x-3)^{3/2} + C}}$$

Ex $\int 7xe^{x^2}\,dx = ?$

Set $u = x^2$.

Then $e^{x^2} = e^u$

and $\frac{du}{dx} = 2x$, so $du = 2x\,dx$

$\frac{1}{2}du = x\,dx$ or $\frac{1}{2x}du = dx$

or $\frac{1}{2\sqrt{u}}du = dx$

$$\text{so} \int 7x e^{x^2} dx = 7 \int e^{x^2} \cdot x\, dx$$

$$= 7 \int e^u \cdot \tfrac{1}{2} du$$

$$= \frac{7}{2} \int e^u du = \frac{7}{2} e^u + C = \underline{\underline{\frac{7}{2} e^{x^2} + C}}$$

Ex $\displaystyle \int \frac{x^2 + 16x + 8}{\sqrt{\frac{x}{2}+1}} dx = ?$

Try $u = \frac{x}{2} + 1 \longrightarrow 2u = x + 2$

$2u - 2 = x$

$du = \frac{1}{2} dx$

$2du = dx$

$$\int \frac{(2u-2)^2 + 16(2u-2) + 8}{\sqrt{u}} \cdot 2\, du$$

$$= 2 \int \frac{4u^2 - 8u + 4 + 32u - 32 + 8}{\sqrt{u}} du$$

$$= 2 \int \frac{4u^2 + 24u - 20}{\sqrt{u}} du$$

$$= 8 \int \frac{u^2 + 6u - 5}{\sqrt{u}} du$$

$$= 8 \int u^{3/2} + 6u^{1/2} - 5u^{-1/2} du$$

$$= \cdots = \underline{\underline{\frac{4}{5}\sqrt{\frac{x}{2}+1}\,\left(x^2 + 24x - 56\right)}}$$