Lecture 38

5 Dec 2018

Midterm 3 Fri Dec 7

My office hours: today (W) 3:30–4:30
tomorrow (Th) 11:00–12:00

Work

A definition from physics: (in 1 dim)

if an object moves for a distance Δx,
acted on by a constant force F

($F > 0$ for force pushing in the positive dir
$F < 0$ " " " " negative dir)

then we say that the force does work on the object,

$$W = F \cdot \Delta x$$

Ex To lift a rock weighing 1 kg (on Earth)
for a height $\Delta x = \frac{1}{2}\,m$ (with constant speed)
we have to exert a force

$$F = mg = (1\,kg)(9.8\,m/s^2) = 9.8\,kg\cdot m/s^2$$

so the work we have to do is

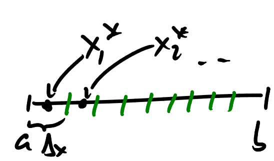
$$W = F\cdot \Delta x = (9.8\,kg\cdot m/s^2)\cdot(\tfrac{1}{2}\,m)$$
$$= 4.9\,kg\cdot m^2/s^2 = \underline{\underline{4.9\,J}}$$

What if the force is not constant?
Doesn't make sense to write $W = F\cdot\Delta x$

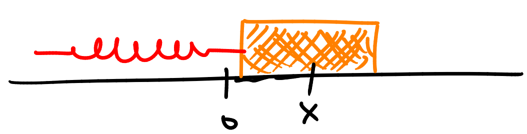
Instead, $W = \int F\,dx$

(One way to think about this: break the process up into many sub-processes,

$$W \approx \sum F(x_i^*)\Delta x \rightsquigarrow W = \int F\,dx)$$



Ex A block is attached to a spring



When the block is at position x
the spring exerts a force $F = -kx$ (spring constant)
("Hooke's Law")

If $k = 2\,\frac{N}{m}$, what is the work done by the spring on the block as it moves from $x=0$ to $x=.03$ m?

$$W = \int_0^{0.03} F\,dx = \int_0^{0.03} (-2)x\,dx = -x^2\Big|_0^{0.03} = \underline{\underline{-.0009\ J}}$$

Why do we want to calculate the work?

$\underline{\text{Because}}$:

$$\underset{\substack{\uparrow\\ \text{net force}}}{F} = \underset{\substack{\uparrow\\ \text{mass}}}{m}\,\underset{\substack{\uparrow\\ \text{acceleration}}}{a}$$

Total work

$$W = \int_{x_0}^{x_1} F\,dx$$

$$= \int_{x_0}^{x_1} ma\,dx$$

$$= \int_{x_0}^{x_1} m\frac{dv}{dt}\,dx$$

$$= \int m\frac{dx}{dt}\,dv$$

$$= \int_{v_0}^{v_1} m\cdot v\cdot dv$$

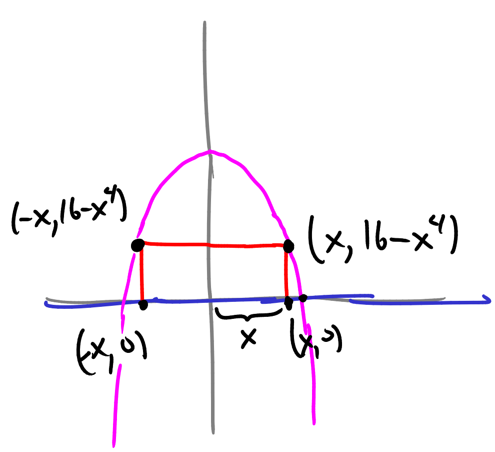
$$= \frac{1}{2}mv^2\Big|_{v_0}^{v_1} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \underline{\text{net change}} \text{ in } \frac{1}{2}mv^2$$

$\uparrow$ $\underline{\underline{\text{kinetic energy}}}$

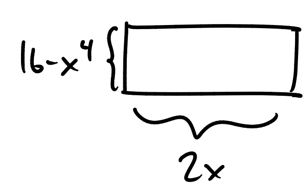
$$\left(\begin{array}{l} x = \text{position} \\ v = \text{velocity} = \frac{dx}{dt} \\ a = \text{acceleration} = \frac{dv}{dt} \end{array}\right)$$

$\underline{\text{Optimization}}$

What is the area of the largest rectangle we can inscribe between the graph $y = 16 - x^4$ and $y = 0$?



area $= 2x(16-x^4)$



So, maximize $A=2x(16-x^4)$ for $x\in[0,2]$

$=32x-2x^5$

crit pts: $\frac{dA}{dx}=32-10x^4=0$

$32=10x^4$

$\frac{16}{5}=x^4 \qquad x=\sqrt[4]{\frac{16}{5}}=\frac{2}{\sqrt[4]{5}} \qquad A=32x-2x^5$

$\int_0^{3/2} 2f(4x)\,dx=\,?$

$u=4x$

$du=4\,dx \qquad \frac{du}{4}=dx$

$\int_{u=0}^{u=6} 2\,f(u)\cdot\frac{du}{4}$

$=\frac{1}{2}\int_0^6 f(u)\,du=\frac{1}{2}(18)=9$

$\int_0^6 f(x)\,dx=18$

$\int_0^6 f(t)\,dt=$

$\int_0^1 x^2\,dx=\frac{1}{3}$

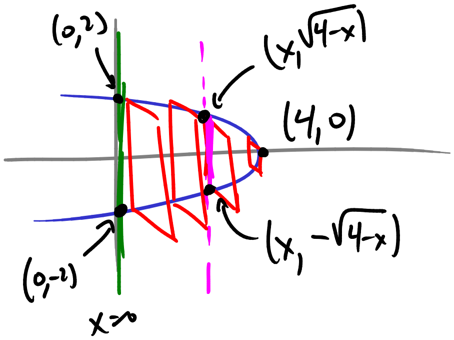
$\int_0^1 t^2\,dt=\frac{1}{3}$

$\int_0^1 u^2\,du=\frac{1}{3}$

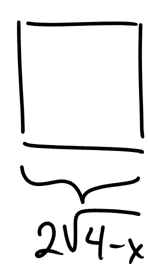
Calc the volume of an object whose base is the region between $x=4-y^2$ and $x=0$

$4-x=y^2$

$y=\sqrt{4-x}$



and whose cross-sections at fixed x are squares.



area $=4\cdot(4-x)$

$V=\int_0^4 A(x)\,dx$

$=\int_0^4 4\cdot(4-x)\,dx$

$=\ldots$

$\sqrt[4]{75}$ by Newton's Method: start with $\sqrt[4]{81} = 3$

$x^4 = 75$ $\quad x^4 - 75 = 0$

look at the function $f(x) = x^4 - 75$ want to solve $f(x) = 0$

Initial guess: $x_0 = 3$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

next guess: $x_1 = 3 - \dfrac{f(3)}{f'(3)}$

here $f(3) = 81 - 75 = 6$

$f'(3) = 4 \cdot 3^3 = 4 \cdot 27 = 108$

$= 3 - \dfrac{6}{108}$

$= 3 - \dfrac{1}{18}$

$f'(x) = 4x^3$