Welcome! M408L, Spring 2010. Integral Calculus.

1st day handout: http://www.ma.utexas.edu/courses/m408l/handout.php

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Lectures MWF 3-4p, WRW 102. [Begin at 3:00, end at 3:50 ± 2 min.]

Discussion TTh

56585	2-3p	WRW 113
56590	3:30-4:30p	RLM 5.124
56595	5-6p	RLM 5.116

Office hours:

me	MF	1:30-2:30p	RLM 9.134
James	MW	11:00a-12:30p	RLM 10.146

Broadly the same format as 408K:

- Homework via QUEST at https://quest.cns.utexas.edu/ 10%
  Due 3am each Mon night / Tue morning
  One extra review assignment, due 3am Friday Jan 29
  Worst 3 (of 15) dropped in grade
  Working together strongly recommended!
- 3 midterm exams (2 hr, evening) 60%
  Feb 23, Apr 6, May 4
- Final exam 30%
  Date unknown until late in the semester!
  Could be anytime during the exam period.

Significantly harder than 408K $\Rightarrow$ Have to work harder to get the same grade.

Many resources available: Office hours
UT Learning Center (Jester A332A)
Your fellow students!
Textbook

Slides available at http://www.ma.utexas.edu/users/neitzke
(+ on Blackboard, <u>soon</u>)

Antiderivatives

Recall the derivative: $f(x) \rightsquigarrow f'(x)$

Examples:
- $f(x) = x^2 \rightsquigarrow f'(x) = 2x$
- $f(x) = \sin(x^3) \rightsquigarrow f'(x) = 3x^2\cos(x^3)$

Suppose we wanted to "go backwards":

$$f \underset{\text{antiderivative}}{\overset{\text{derivative}}{\rightleftarrows}} f'$$

Definition: $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Any function $f(x)$ has many antiderivatives!

Ex: $f(x) = x$ has antiderivatives $F(x) = \frac{1}{2}x^2$

$F(x) = \frac{1}{2}x^2 + 7$

$F(x) = \frac{1}{2}x^2 - 32$

$\vdots$

To get all possible antiderivatives of $f(x)$, first find one particular antiderivative, and then add an arbitrary constant. (usually called C)

Ex:
- $f(x) = \cos x$ has general antiderivative $F(x) = \sin x + C$
- $f(x) = x^n$ " " " $F(x) = \frac{1}{n+1}x^{n+1} + C$ $\boxed{\text{EXCEPT when } n=-1}$

Build more complicated examples from these simple ones:

- $f(x) = 9x^2 + 6x^{3/2} - \frac{2}{x^4} + \cos x$ $\qquad \frac{1}{x^4} = x^{-4}$

  has general antiderivative

  $F(x) = 9\left(\frac{1}{3}x^3\right) + 6\left(\frac{1}{(5/2)}x^{5/2}\right) - 2\left(\frac{1}{-3}x^{-3}\right) + \sin x + C$

  $= 3x^3 + \frac{12}{5}x^{5/2} + \frac{2}{3}x^{-3} + \sin x + C$

- $f(x) = \cos 2x$ has general antideriv
  $F(x) = \frac{1}{2}\sin 2x + C$

- Is $F(x) = \frac{4}{3}\ln(1+3x)$ an antiderivative of $f(x) = \dfrac{4}{1+3x}$?
  $$F'(x) = \frac{4}{3}\cdot\frac{1}{1+3x}\cdot 3 = \frac{4}{1+3x} \quad \text{— } \underline{\text{YES}}$$

Sometimes we don't want the <u>most general</u> antideriv, we want some <u>specific</u> one:

- What is the function $f(x)$ which has $f'(x) = 4x+7$?
  <u>AND</u> $f(1) = 6$

Since $f'(x) = 4x+7$, $f(x) = 2x^2+7x+C$

Since $f(1) = 6$,
$$2+7+C = 6$$
$$9+C = 6$$
$$C = -3$$

So $\underline{f(x) = 2x^2+7x-3}$

<u>Why care</u> about antiderivatives?

A Standard reason:

$$\begin{tikzcd} \text{position } s(t) \arrow[r, bend left, "\text{derivative}"] & \text{velocity } v(t) \arrow[l, bend left, "\text{antideriv.}"] \arrow[r, bend left, "\text{derivative}"] & \text{acceleration } a(t) \arrow[l, bend left, "\text{antideriv.}"] \end{tikzcd}$$

- A train accelerates with constant accel. $a(t) = 4 \text{ ft}/\text{s}^2$

At time $t=0$ it has velocity $v(t=0) = 100$ ft/s

and position $s(t=0) = 0$ ft.

How far does it go in 20 s? $s(t=20\text{ s}) = ?$

$$\left.\begin{aligned} &a(t) = 4 \\ &v(t) = 4t + C \\ &v(t=0) = 100 \end{aligned}\right] \Longrightarrow C = 100$$

so $v(t) = 4t + 100$

$$\left.\begin{aligned} &s(t) = 2t^2 + 100t + D \\ &s(t=0) = 0 \end{aligned}\right] \Rightarrow D = 0$$

so $s(t) = 2t^2 + 100t$

so $s(t=20) = 2(20^2) + 100(20) = \underline{\underline{2800 \text{ ft}}}$