Lecture 32

9 Apr 2010

Last time: <u>series</u> $a_1 + a_2 + a_3 + \cdots$

$$= \sum_{i=1}^{\infty} a_i$$

[Can also have seq. beginning from $i=0$ instead of $i=1$
Then have series like $a_0 + a_1 + a_2 + \cdots$

$$= \sum_{i=0}^{\infty} a_i \quad]$$

<u>Partial sums</u> of $\sum_{i=1}^{\infty} a_i$ are $s_n = \sum_{i=1}^{n} a_i$

$s_1 = a_1$
$s_2 = a_1 + a_2$
$s_3 = a_1 + a_2 + a_3$
$\vdots$

A common class of examples: <u>geometric series</u> $\sum_{i=1}^{\infty} ar^{i-1}$

[or $\sum_{i=0}^{\infty} ar^i$]

<u>Partial sums</u> of $\sum_{i=1}^{\infty} ar^{i-1}$ are $s_n = a \cdot \frac{1-r^n}{1-r}$

Taking $n \to \infty$ limit:

$$\sum_{i=1}^{\infty} ar^{i-1} \begin{cases} \underline{\text{converges}} \text{ to } \frac{a}{1-r} & \text{if } |r| < 1 \\ \underline{\text{diverges}} & \text{if } |r| \geq 1 \end{cases}$$

Ex Does $\sum_{i=0}^{\infty} \frac{\pi^i}{3^{i+1}}$ converge?

$$\sum_{i=0}^{\infty} \frac{\pi^i}{3^{i+1}} = \sum_{i=0}^{\infty} \frac{\pi^i}{3 \cdot 3^i} = \sum_{i=0}^{\infty} \frac{1}{3}\left(\frac{\pi}{3}\right)^i$$

geometric: $a = \frac{1}{3}$, $r = \frac{\pi}{3}$

Since $|r| = \frac{\pi}{3} \approx \frac{3.1416}{3} > 1$, this geometric series <u>diverges</u>.

Ex Consider the repeating decimal

$$1.\overline{73} = 1.73737373\ldots$$

Write it as a fraction.

$$1.73737373\ldots$$

$$= 1 + 0.73 + 0.0073 + 0.000073 + \cdots$$

$$= 1 + \underbrace{\frac{73}{100} + \frac{73}{100^2} + \frac{73}{100^4} + \cdots}_{\text{geometric series with } a = \frac{73}{100},\ r = \frac{1}{100}}$$

$$\left[\sum_{i=1}^{\infty} \frac{73}{100} \cdot \left(\frac{1}{100}\right)^{i-1}\right]$$

$$= 1 + \frac{a}{1-r} = 1 + \frac{73/100}{1 - 1/100} = 1 + \frac{73/100}{99/100} = 1 + \frac{73}{99} = \frac{172}{99}$$

Ex Consider the series

$$\sum_{i=0}^{\infty} \frac{(x+3)^i}{2^i}$$

For what values of x does it converge?

$$\sum_{i=0}^{\infty} \left(\frac{x+3}{2}\right)^i$$

geometric with $a=1$

$r = \frac{x+3}{2}$

The series will converge when $|r| < 1$.

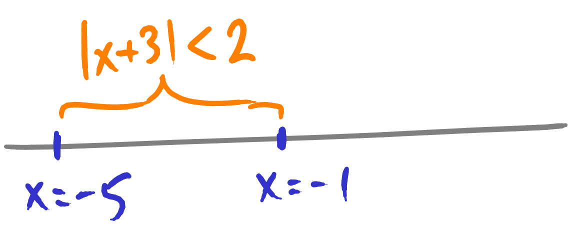
ie $\left|\frac{x+3}{2}\right| < 1$

$|x+3| < 2$

To find the boundaries of this region on the # line, solve $|x+3| = 2$

ie $x+3=2$ or $x+3=-2$

$x=-1$ or $x=-5$



So the series converges for $\underline{\underline{-5 < x < -1}}$.

Ex $\displaystyle\sum_{i=1}^{\infty} \frac{(\cos x)^i}{2^i}$

For what x does this series converge?

$$\sum_{i=1}^{\infty} \left(\frac{\cos x}{2}\right)^i$$

geometric: $a = \dfrac{\cos x}{2}$

$r = \dfrac{\cos x}{2}$

$$\left[\sum_{i=1}^{\infty} \left(\frac{\cos x}{2}\right)\cdot\left(\frac{\cos x}{2}\right)^{i-1}\right]$$

Converges if $|r|<1$ i.e. $\left|\dfrac{\cos x}{2}\right| < 1$

i.e. $|\cos x| < 2$

That's true for <u>all</u> x: so this series <u>converges for all x</u>.

$\left[\text{to } \dfrac{a}{1-r}\right]$

Test For Divergence:

If $\lim_{i\to\infty} a_i$ <u>doesn't exist</u>, or if it exists but it's not 0,

then $\displaystyle\sum_{i=1}^{\infty} a_i$ <u>diverges</u>.

Ex $1+2+3+4+\cdots = \displaystyle\sum_{i=1}^{\infty} i$ <u>diverges</u> (b/c $\lim_{i\to\infty} i$ doesn't exist)

Ex $\displaystyle\sum_{i=1}^{\infty} \frac{3i+4}{4i-7}$ <u>diverges</u> (b/c $\displaystyle\lim_{i\to\infty} \frac{3i+4}{4i-7} = \frac{3}{4} \neq 0$)

Ex $\sum_{i=1}^{\infty} \frac{1}{i}$: $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ might converge

$\left(\lim_{i\to\infty} \frac{1}{i} = 0\right)$

- this test gives no info. here

Ex $\sum_{i=1}^{\infty} \frac{1}{i^2}$: $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots$ might converge

$\left(\lim_{i\to\infty} \frac{1}{i^2} = 0\right)$

Ex $\sum_{i=0}^{\infty} \tan^{-1}(i)$: diverges $\left(\text{since } \lim_{i\to\infty} \tan^{-1}(i) = \frac{\pi}{2} \neq 0\right)$

In fact: $\sum_{i=1}^{\infty} \frac{1}{i}$ diverges

$\sum_{i=1}^{\infty} \frac{1}{i^2}$ converges $\qquad \left(\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}\right)$

We'll see why, in the next lecture...