Review:

$$\int f(x)\,dx$$

this means a function $F(x)$ with $F'(x) = f(x)$

"antiderivative", "indefinite integral"

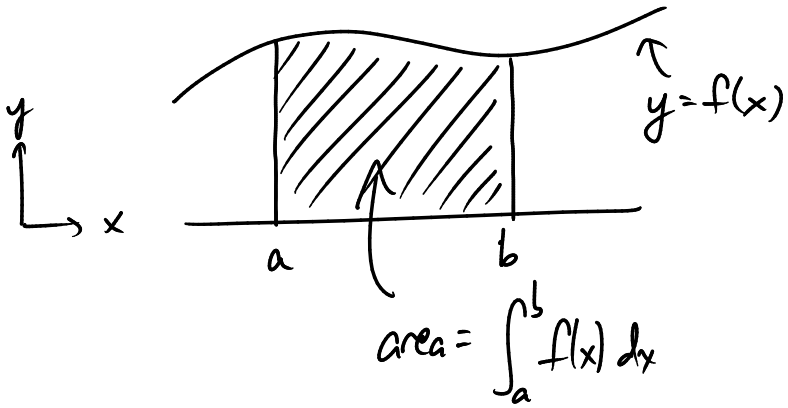
$$\int_a^b f(x)\,dx$$

this means the definite integral of $f(x)$ from a to b

Ex $\int x\,dx = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 + 2$

$= \frac{1}{2}x^2 + C$



FTC says these 2 things are related!

FTC ① $\frac{d}{dx}\left(\int_c^x f(t)\,dt\right) = f(x)$ i.e. $\int_c^x f(t)\,dt$ is an antiderivative of $f(x)$

Q What is the derivative of 1) $F(x) = \int_4^x \sin t\,dt$? i.e. $\frac{dF}{dx} = ?$

2) $F(x) = \int_2^x \sin t\,dt$?

1) $\sin t$ $\sin x$ $\sin x + C$

2) $\sin x$ $\sin x + C$

Q What is the deriv. of 1) $F(x) = \int_{17834}^{x^2} \frac{1}{1+t}\,dt$?

2) $F(x) = \int_3^{\ln(1+2x)} \sqrt{1+t^2}\,dt$?

1) $F'(x) = \dfrac{2x}{1+x^2}$ ⟶ why? $\displaystyle\int_{17834}^{x^2} \frac{1}{1+t}\,dt$

2) $F'(x) = \dfrac{2\sqrt{1+\ln(1+2x)^2}}{1+2x}$:

We know $\dfrac{d}{du}\displaystyle\int_{17834}^{u} \frac{1}{1+t}\,dt$

$= \dfrac{1}{1+u}$

So, make substitution $u = x^2$

then $\dfrac{d}{dx}\displaystyle\int_{\_}^{u} \frac{1}{1+t}\,dt$

$= \left(\dfrac{du}{dx}\right) \cdot \dfrac{d}{du}\displaystyle\int_{\_}^{u} \frac{1}{1+t}\,dt$

$= \left(\dfrac{du}{dx}\right) \cdot \dfrac{1}{1+u} = 2x \cdot \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}\left[\displaystyle\int_3^{\ln(1+2x)} \sqrt{1+t^2}\,dt\right]$

$= \dfrac{du}{dx} \cdot \dfrac{d}{du}\displaystyle\int_3^{u} \sqrt{1+t^2}\,dt$

$= \dfrac{2}{1+2x} \cdot \sqrt{1+u^2}$

$= \dfrac{2}{1+2x} \cdot \sqrt{1+\ln(1+2x)^2}$

$u = \ln(1+2x)$

$\dfrac{du}{dx} = \dfrac{d}{dx}\ln(1+2x)$

$= \dfrac{2}{1+2x}$

Q What are the critical points of $F(x) = \displaystyle\int_5^{x} \frac{2-t}{1+t^3}\,dt$?

(or at least their x-values)

A: $x = 2$ $x = -1$

why? critical pts are pts where $F'(x) = 0$ or undefined of F

$$F'(x) = \frac{2-x}{1+x^3}$$

$F'(x) = 0$ just if $2 - x = 0$

$\boxed{x = 2}$

$F'(x)$ undefined if $1 + x^3 = 0$

$x^3 = -1$

$\boxed{x = -1}$

Q 1) What is $\frac{d}{dx}\left(\int_x^5 \tan(t^3)\, dt\right)$?

2) What is $\frac{d}{dx}\left(\int_x^{2x} \ln t\, dt\right)$?

$f(v)\, dv$

$\tan(5^3)(0) - \tan(x^3)$

$-\tan(x^3)$

1) $\frac{d}{dx}\left(\int_x^5 \tan(t^3)\, dt\right) = \frac{d}{dx}\left(-\int_5^x \tan(t^3)\, dt\right) = -\tan(x^3)$

FTC 2 $\int_a^b f(x)\, dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$

Q $\int_2^4 x^3\, dx = ?$ 60? 36? 63? 56?

an antideriv. of x^3 is $F(x) = \frac{1}{4}x^4$

$F(4) - F(2) = \frac{4^4}{4} - \frac{2^4}{4} = 64 - 4 = \underline{60}$

Some useful antiderivs

$f(x)$	$F(x)$	
x	$\frac{1}{2}x^2$	
$\cos x$	$\sin x$	
$\sin x$	$-\cos x$	
$\frac{1}{x}$	$\ln x$	
e^x	e^x	
$\sec^2 x$	$\tan x$	
x^n	$\frac{x^{n+1}}{n+1}$	(if $n \neq -1$)