Lecture 2

Admin info:

you can now see solutions to HW01 in Quest

HW01 was due 3am today — Learning Modules each M, W, Sat night midnight

HW02 is due 3am next Tue

lowest 6 QR (discussion) grades will be dropped

My office hr M 2-3

Q is $\int (1+\sin x)^3\,dx = \frac{1}{4}(1+\sin x)^4 + C$? NO

is $\int x^7\,dx = \frac{1}{8}x^8 + C$? YES

$\int x^{-1/3}\,dx = \frac{3}{2}x^{2/3} + C$? YES

Some useful antiderivatives

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \left(\neq \frac{1}{\sin x}\right)$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

(see table on p. 403 of text for more)

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \frac{d}{dx}\sin x - \sin x \frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{(\cos x)^2 - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x \checkmark$$

Q 1) What is $\int u^{2/3}\,du$? (write the most general antiderivative)

2) Find a function $F(u)$ with $\frac{dF}{du} = u^{2/3}$ and $F(1) = 1$.

1) $\frac{3}{5}u^{5/3} + C$

2) $F(u) = \frac{3}{5}u^{5/3} + C$

$F(1) = 1$ means

$\frac{3}{5}(1)^{5/3} + C = 1$

$\frac{3}{5} + C = 1$

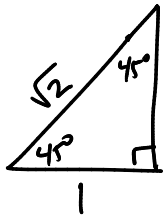
$C = 2/5$ so $F(u) = \frac{3}{5}u^{5/3} + \frac{2}{5}$

Q What is $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$?

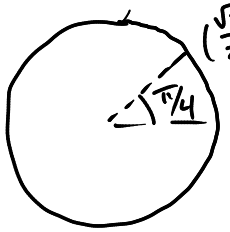
$$\tan^{-1}x \Big]_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$$

$\tan^{-1}(1) \neq \frac{1}{\tan(1)}$ if $\tan^{-1}(1) = a$

$\neq \frac{\sin^{-1}(1)}{\cos^{-1}(1)}$ then $1 = \tan(a)$



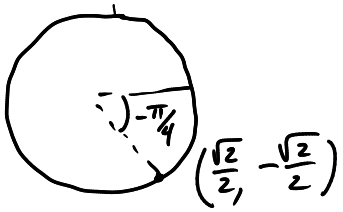
$\tan(45°) = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1$



$\tan(\pi/4) = 1$

in radians, this says $\tan\left(\frac{\pi}{4}\right) = 1$

now for $\tan^{-1}(-1)$? $\tan^{-1}(-1) = -\frac{\pi}{4}$

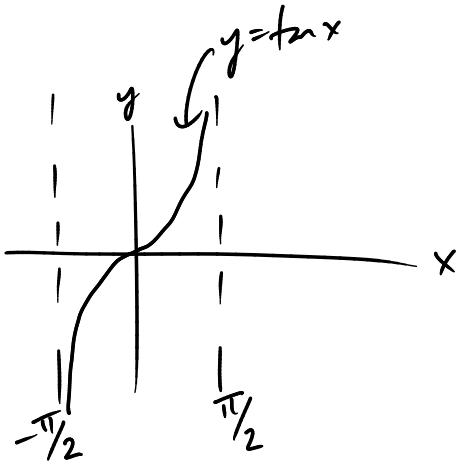


another way: $\tan(x) = -\tan(-x)$ (because $\sin(-x) = -\sin(x)$, $\cos(-x) = \cos(x)$)

so if know $\tan\left(\frac{\pi}{4}\right) = 1$ then also know $\tan\left(-\frac{\pi}{4}\right) = -1$

So: $\tan^{-1}(x)\Big]_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$

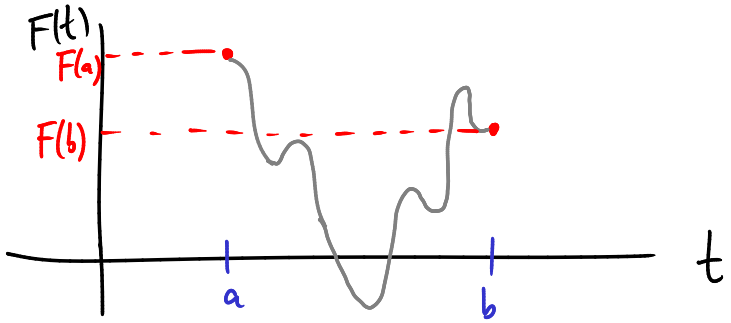
$= \left(\frac{\pi}{4}\right) - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$



Net change theorem

Suppose have a function $F(t)$. $\frac{dF}{dt} = F'(t)$ = the rate of change of $F(t)$,

$\int_a^b F'(t) = F(b) - F(a)$ = net change of $F(t)$ as t goes from a to b



Q Water flows into a reservoir at the rate $(10t+6)$ ft^3/s. (t measured in s)
The reservoir contains 400 ft^3 of water at time $t=0$.
How much does it contain at $t=10$?

$$\int_0^{10} 10t+6\,dt = F(10) - F(0)$$

$$\int_0^{10} 10t+6 = F(10) - 400$$

$$\frac{10(10)^2}{2} + 6(10) = F(10) - 400$$

$$\downarrow$$

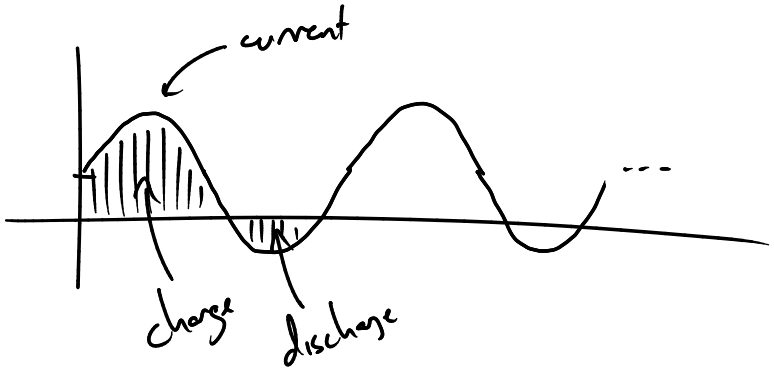
$$560 + 400 = 960 = F(10)$$

Q A rechargeable battery is connected to a load which can charge or discharge it.

The current flowing into the battery is $\sin(\pi t) + \frac{1}{2}$ (units of charge)/day.

The battery starts with 10 units charge at $t = 0$

How much does it have at $t = 6$?



[Rk current is the derivative of charge!]

$$\int_0^6 \left(\sin(\pi t) + \tfrac{1}{2}\right) dt = F(6) - F(0)$$

so $$F(6) = F(0) + \int_0^6 \left(\sin(\pi t) + \tfrac{1}{2}\right) dt$$

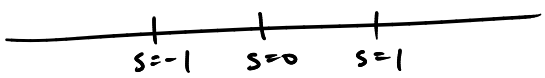
$$= 10 + \left[-\frac{\cos(\pi t)}{\pi} + \frac{1}{2}t \right]_0^6$$

$$= 10 + \left(-\tfrac{1}{\pi} + 3\right) - \left(-\tfrac{1}{\pi} + 0\right) = 10 + \left(3 - \tfrac{1}{\pi}\right) - \left(-\tfrac{1}{\pi}\right)$$

$$= \underline{13}$$

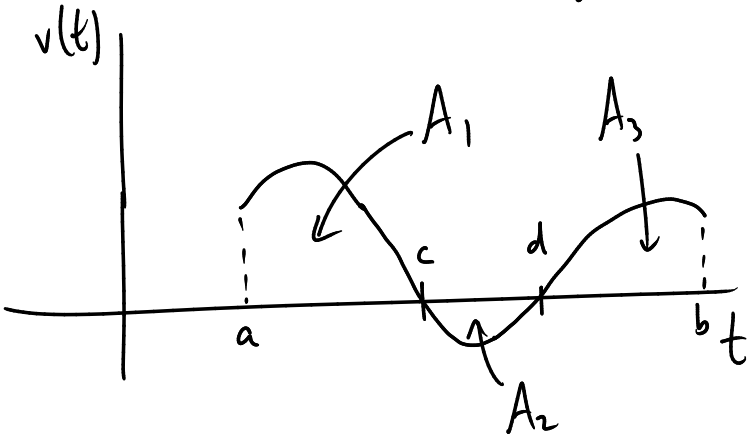
Total displacement

Recall: if $s(t)$ = position (along some line)



$v(t) > 0$: $s(t)$ increasing i.e. moving to the right
$v(t) < 0$: $s(t)$ decreasing i.e. moving to the left

$s'(t) = v(t)$ = velocity



total displacement

$$s(b) - s(a) = \int_a^b v(t)\,dt$$

$$= A_1 - A_2 + A_3$$

(how far to the right we travel between time a and time b)

total distance (how many miles did the odometer reading increase) $= A_1 + A_2 + A_3$

$$= \int_a^b |v(t)|\,dt$$

Ex A particle moves along a line with $v(t) = t^2 - t - 6$ m/s (t in s) from time $t=1$ to $t=4$.

a) What is the **total displacement**?

b) What is the **total distance**?

a) total displ $= s(4) - s(1) = \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6\,dt$

$$= \cdots = -\frac{9}{2} \text{ m}$$

(i.e. $\frac{9}{2}$ m **left**)

b) $\int_1^4 |v(t)|\,dt$ $\qquad v(t) = (t-3)(t+2)$

so $\int_1^4 |v(t)|\,dt$

$= -\int_1^3 v(t)\,dt + \int_3^4 v(t)\,dt = \cdots = \frac{61}{6}\ m$