Lecture 2

Administration:

HW01 due today 3am
HW02 due next Tue 3am
⋮

Learning Modules due every M, W, Sat night (midnight)

We'll drop lowest 6 QR problems

My office hr M 2-3pm
W afternoon

Tom Calcles M 5-6pm

Q 1) is $\int (1+\sin x)^3\,dx = \frac{1}{4}(1+\sin x)^4 + C$? NO

2) is $\int x^7\,dx = \frac{1}{8}x^8 + C$? YES

ii $\int x^{-1/3}\,dx = \frac{3}{2}x^{2/3} + C$? YES

Some useful antiderivatives ← see table on p. 403 of text

$$\int e^x\,dx = e^x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

or, $\sin^{-1} x + C$ (not $\frac{1}{\sin x} + C$)

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

or, $\tan^{-1} x + C$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}\left((\cos x)^{-1}\right) = -(\cos x)^{-2}\cdot(-\sin x) = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x \tan x \checkmark$$

Q 1) What is $\int u^{2/3}\,du$? the most general antideriv

2) Find a function $F(u)$ with $\frac{dF}{du} = u^{2/3}$ and $F(1) = 1$.

1) $\frac{3}{5}u^{5/3} + C$

2) $\frac{dF}{du} = u^{2/3}$ so $F(u) = \frac{3}{5}u^{5/3} + C$

and $F(1) = 1$, so

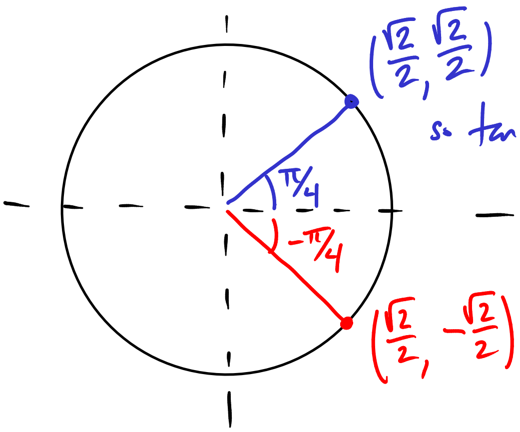
$\frac{3}{5} + C = 1$, so $C = \frac{2}{5}$

so $\underline{\underline{F(u) = \frac{3}{5}u^{5/3} + \frac{2}{5}}}$

Q What is $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$?

$$\tan^{-1}x \Big]_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \underline{\underline{\frac{\pi}{2}}}$$



so $\tan\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \underline{\underline{1}}$ why?

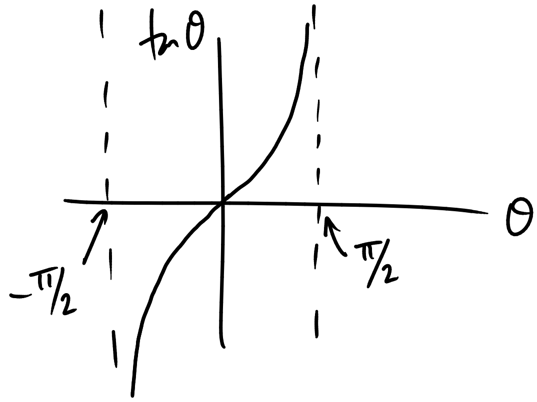
so $\tan\left(-\frac{\pi}{4}\right) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \underline{\underline{-1}}$

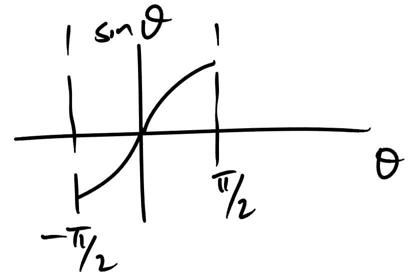
Rk Actually there are lots of angles θ with $\tan(\theta) = 1$.

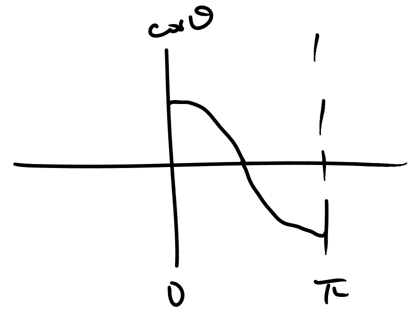
$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, -\frac{3\pi}{4}, \ldots$$

The definition of $\tan^{-1}$ says always pick a θ between $-\pi/2$ and $\pi/2$.

(For $\sin^{-1}$ take θ between $-\pi/2$ and $\pi/2$
$\cos^{-1}$ take θ —— 0 and π)



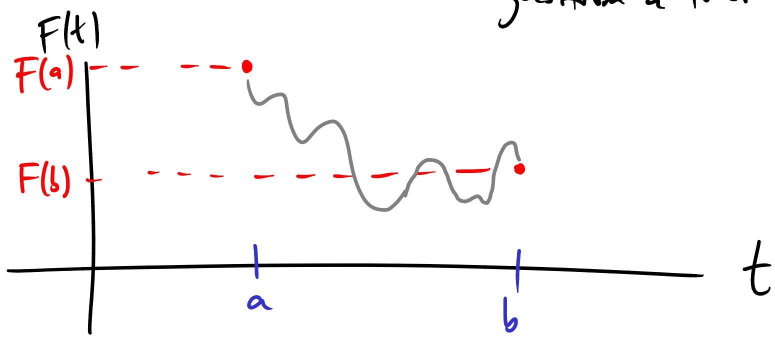




Net change

Suppose we have a function $F(t)$ $\quad \frac{dF}{dt} = F'(t) =$ the rate of change of F.

$$\int_a^b F'(t)\,dt = F(b) - F(a) = \text{net change of } F(t) \text{ as } t \text{ goes from } a \text{ to } b.$$



Q Sand is flowing into a basin at the rate $(10t+6)\ ft^3/s$

(t measured in s)

The basin contains $400\ ft^3$ of sand at time $t=0$.

How much does it contain at time $t=10$?

$$\text{total sand} = \text{initial} + \int_0^{10} (\text{rate})\,dt$$

$$= 400 + \int_0^{10} (10t+6)\,dt$$

$$= 400 + \left(\frac{10t^2}{2} + 6t\right)\Big]_0^{10}$$

$$= 400 + (5t^2 + 6t)\Big]_0^{10}$$

$$= 400 + \left[(5(10)^2 + 6(10)) - (0)\right]$$

$$= 400 + (5(100) + 60)$$

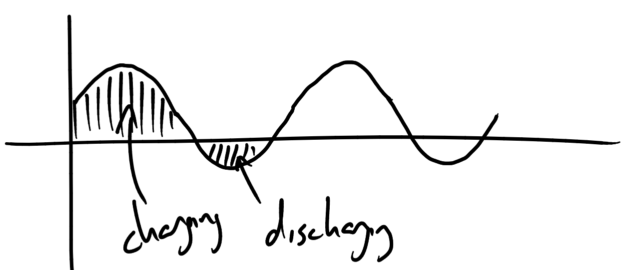
$$= 400 + 500 + 60$$

$$= 960 \text{ ft}^3$$

Q A rechargeable battery is connected to a load that can charge or discharge it. The current flowing into it is

$$\sin(\pi t) + \frac{1}{2} \qquad (t \text{ in days})$$

The battery starts with 10 units of charge at $t=0$. How much does it have at $t=6$? (after 6 days)



$F(t) =$ charge at time t

$$F(6) - F(0) = \int_0^6 \left(\sin(\pi t) + \frac{1}{2}\right) dt$$

$$= -\frac{\cos(\pi t)}{\pi} + \frac{t}{2}\Big]_0^6$$

$$= \left(-\frac{1}{\pi} + \frac{6}{2}\right) - \left(-\frac{1}{\pi} + 0\right)$$

$$\left(\frac{d}{dt}\,\frac{-\cos(\pi t)}{\pi} = \frac{+\pi \sin \pi t}{\pi} = \sin \pi t\right)$$

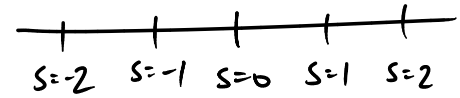
$$= 3$$

$$F(6) - 10 = 3$$

$$F(6) = \underline{\underline{13}}$$

Total displacement vs total distance

Recall: if $s(t)$ = position (along a line)

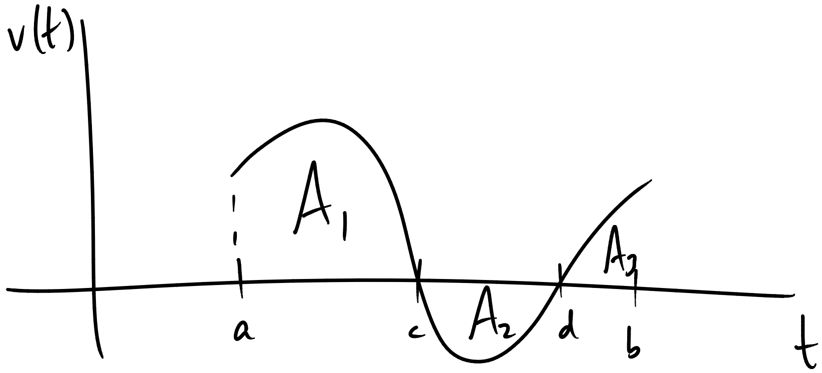


$$s'(t) = v(t) = \text{velocity}$$

$v(t) > 0$: moving to the right

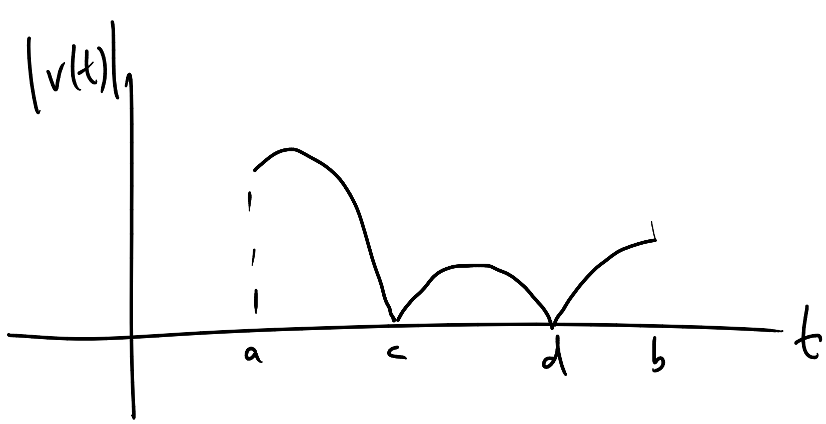
$v(t) < 0$: moving to the left

speedometer shows speed: speed $= |v(t)|$



total displacement $\quad s(b) - s(a) = \int_a^b v(t)\,dt = A_1 - A_2 + A_3$

total distance $\quad \int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$



Q A particle moves along a line with $v(t) = t^2 - t - 6$ m/s (t in s) from time $t=1$ to $t=4$.

a) What is the total displacement?

b) What is the total distance?

a) displacement $= \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6 \, dt$

$\quad\ = \cdots = -\frac{9}{2}$ m

$\quad \ (\frac{9}{2}$m to the left$)$

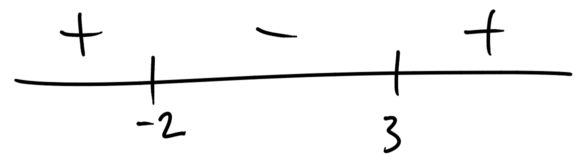
$\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \parallel$

$\ \ \ \ \ \ \ \ \ \ s(4) - s(1)$

b) total distance $= \int_1^4 |v(t)|\,dt$

make a "sign chart" for $v(t)$:

$v(t) = (t-3)(t+2)$



use that to get $\int |v(t)|\,dt$