# Lecture 3

Admin: next LM due Sat night midnight
HW02 due Tue 3am

I have office hr today 5-6 RLM 9.134
M 2-3

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## Method of substitution ("u-substitution")

A method of finding antiderivatives.

Ex $\int \sqrt{2x-3}\,dx = ?$

Try to relate this to something easier to understand: introduce $u = 2x-3$

Eliminate x in favor of u.

$$\int \sqrt{2x-3}\,dx = \int \sqrt{u}\,dx$$

$$\int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C$$

Need to relate dx to du. $\frac{du}{dx} = 2$, so $du = 2\,dx$

$\frac{1}{2}du = dx$

$$\text{So } \int \cdots = \int \sqrt{u} \cdot \left(\tfrac{1}{2}du\right) = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + C\right) = \frac{1}{3}u^{3/2} + C' = \frac{1}{3}(2x-3)^{3/2} + C'$$

Q $\int 7x e^{x^2}\,dx = ?$

try $u = x^2$

$\frac{du}{dx} = 2x$ so $du = 2x\,dx$

or $\frac{du}{2x} = dx$

$\int e^{-x^2}\,dx = ?$

can't do by substitution!!

erf(x)

then $\int 7x e^{x^2}\,dx = 7\int x e^{u} \frac{du}{2x} = \frac{7}{2}\int e^{u}\,du = \frac{7}{2}e^{u} + C$

$= \frac{7}{2}e^{x^2} + C$

or: $u = e^{x^2}$ $\quad \frac{du}{dx} =$

$du = 2xe^{x^2}\,dx$

$\int \frac{7}{2}\,du = \frac{7}{2}u + C = \frac{7}{2}e^{x^2} + C$

Ex $\int \frac{x^2+16x+8}{\sqrt{\frac{x}{2}+1}}\,dx = ?$

$u = \frac{x}{2} + 1$

$u = \sqrt{\frac{x}{2}+1}$

$u = x^2 + 16x + 8$

$u = \left(\frac{x}{2}+1\right)^{-1/2}$

$du = -\frac{1}{4}\left(\frac{x}{2}+1\right)^{-3/2}dx$

looks hard

$u = \frac{x}{2} + 1$

$du = \frac{1}{2}\,dx$ $\qquad 2\,du = dx$

$\int \frac{x^2+16x+8}{\sqrt{u}}\,2\,du$

need to eliminate x: $u = \frac{x}{2} + 1$

$2u = x + 2$

$2u - 2 = x$

$\longrightarrow$ substitute. $2\int \frac{(2u-2)^2 + 16(2u-2) + 8}{\sqrt{u}}\,du$

$= 2\int \frac{4u^2 - 8u + 4 + 32u - 32 + 8}{\sqrt{u}}\,du$

$= 2\int \frac{4u^2 + 24u - 20}{\sqrt{u}}\,du$

$= 2\int 4u^{3/2} + 24u^{1/2} - 20u^{-1/2}\,du$

do by power rule

sub back $u = \frac{x}{2}+1$

finally get $= \frac{4}{5}\sqrt{\frac{x}{2}+1}\left(x^2+24x-56\right)$

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Q $\int \sin(2x)\,dx = ?$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int \sin(u)\cdot \frac{du}{2}$

$= \frac{1}{2}\int \sin(u)\,du = -\frac{1}{2}\cos(u) + C$

$\int_0^{\pi/2} \sin(2x)\,dx = ?$ 1 1/4 1/2 -1

$u = 2x$

THIS IS NOT $\int_0^{\pi/2} \sin(u)\,\frac{du}{2}$

rather, $\int_{x=0}^{x=\pi/2} \sin(2x)\,dx = \int_{u=0}^{u=\pi} \sin(u)\,\frac{du}{2} = -\frac{1}{2}\cos(u)\Big|_{u=0}^{u=\pi}$

$= -\frac{1}{2}(-1-1) = \frac{1}{2}$

could also do $\sin(2x) = 2\sin x\cos x$

$u = \sin x$

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Ex $\int_{\pi/3}^{\pi/2} (\cos 3x)\,e^{\sin 3x}\,dx$

$u = \sin 3x$

$\frac{du}{dx} = 3\cos 3x$ $\frac{du}{3} = \cos 3x\,dx$

$$= \int_{x=\pi/3}^{x=\pi/2} e^u \frac{du}{3}$$

$$= \frac{1}{3} e^u \Big|_{u=0}^{u=-1} = \frac{1}{3}\left(e^{-1} - e^0\right) = \frac{1}{3}\left(e^{-1} - 1\right)$$

or: $\frac{1}{3} e^{\sin 3x} \Big|_{\pi/3}^{\pi/2} = \cdots = \frac{1}{3}\left(e^{-1} - 1\right)$

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Ex $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

~~$u = \sin x$~~

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= \int -\frac{du}{u}$$

$$= -\ln|u| + C$$

$a = \ln u$

$e^a = u$

$$= -\ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$= \underline{\ln|\sec x| + C}$$

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Q $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}} dx$

$2du = \frac{1}{\sqrt{x}} dx$

$$= \int e^u \cdot 2\, du$$

$$= 2e^u + C = \underline{2e^{\sqrt{x}} + C}$$

(Check: $\frac{d}{dx}\left(2e^{\sqrt{x}}\right) = 2 \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{\sqrt{x}}$)

Q $\displaystyle\int \frac{dx}{\sqrt{x}(1+x)}$

$u=\sqrt{x}$ ?

$u=1+x$ ?

Try $u=1+x$: $du=dx$

$$\int \frac{du}{\sqrt{u-1}\,(u)} \qquad x=u-1$$

not helping

Try $u=\sqrt{x}$: $\dfrac{du}{dx}=\dfrac{1}{2\sqrt{x}}$ $\quad du=\dfrac{1}{2}\dfrac{dx}{\sqrt{x}}$ $\quad 2du=\dfrac{dx}{\sqrt{x}}$

$$\int \frac{2\,du}{1+x} = \int \frac{2\,du}{1+u^2} = 2\tan^{-1}(u) = \underline{\underline{2\tan^{-1}(\sqrt{x})+C}}$$