Lecture 3

Admin: next LM due Sat night midnight
next HW due Tue morning 3am
my office hr Thu 5-6pm RLM 9.134
Mon 2-3 pm
CalcLab is now open!

Method of substitution ("u-substitution")

Another method of finding antiderivatives.

Ex $\int \sqrt{2x-3}\, dx = ?$

Try to replace this by an indef $\int$ that's easier to do: introduce $u = 2x-3$

Then $\int \sqrt{2x-3}\, dx = \int \sqrt{u}\, dx$

To deal with the "dx" part: $\frac{du}{dx} = 2$ i.e. $du = 2\,dx$

$\frac{1}{2} du = dx$

substitute that for dx:

$$\int \sqrt{u}\, dx = \int \sqrt{u}\left(\tfrac{1}{2} du\right)$$

$$= \frac{1}{2}\int \sqrt{u}\, du = \frac{1}{2}\int u^{1/2}\, du = \frac{1}{2}\cdot\frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} u^{3/2} + C$$

$$= \frac{1}{3}(2x-3)^{3/2} + C$$

$$\left(\text{or: } \frac{1}{3}\sqrt{(2x-3)^3} + C\right)$$

Q, ① $\int 7xe^{x^2}\, dx = ?$ ② $\int e^{-x^2}\, dx = ?$

$u = x^2$

$du = 2x\,dx \leftarrow \frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$$\int 7e^{x^2}x\,dx$$
$$= \int 7e^u \cdot \tfrac{1}{2}du$$
$$= \frac{7}{2}\int e^u\,du = \frac{7}{2}e^u + C = \underline{\underline{\frac{7}{2}e^{x^2}+C}}$$

if we try $u = e^x$
no help

if try $u = e^{x^2}$
$du = 2xe^{x^2}dx$
$$\int 7xe^{x^2}dx = \frac{7}{2}\int du$$
$$= \frac{7}{2}u + C = \underline{\underline{\frac{7}{2}e^{x^2}+C}}$$

For $\int e^{-x^2}dx$: the antiderivative exists but there's no formula for it in terms of sin, cos, tan exp, powers, logs, --

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<u>Ex</u> 1) $\int \sin^2\theta\cos\theta\,d\theta = ?$

possibilities:
$u = \sin\theta$
$u = \sin^2\theta$
$u = \sin^3\theta$
$u = \cos\theta$

If take $u = \sin\theta$ $\quad du = \cos\theta\,d\theta$

then $\int \sin^2\theta\cos\theta\,d\theta$
$$= \int u^2\,du = \frac{1}{3}u^3 + C = \underline{\underline{\frac{1}{3}\sin^3\theta + C}}$$

2) $\int \frac{\cos(\ln t)}{t}dt = ?$

$u = \ln t$
$du = \frac{dt}{t}$

$$\int \cos(\ln t)\frac{dt}{t}$$
$$= \int \cos(u)\,du$$
$$= \sin(u) + C$$
$$= \underline{\underline{\sin(\ln t) + C}}$$

<u>Q</u> $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ $\qquad u = \sqrt{x}$ $\qquad du = \frac{1}{2}x^{-1/2}dx$
$$= \frac{1}{2\sqrt{x}}dx$$
so $2du = \frac{dx}{\sqrt{x}}$

$$= \int e^u \cdot 2\,du = 2e^u + C = \underline{\underline{2e^{\sqrt{x}} + C}}$$

Q 1) $\int \sin(2x)\,dx = ?$

$u = 2x$

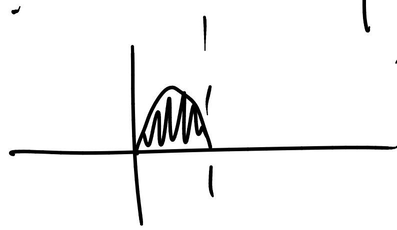
$du = 2\,dx$

$\frac{du}{2} = dx$

$$= \int \sin(u)\,\frac{du}{2}$$

$$= \frac{-\cos(u)}{2} + C = \underline{\underline{\frac{-\cos(2x)}{2} + C}}$$

2) $\int_0^{\pi/2} \sin(2x)\,dx = ?$ $\quad 1,\ 2,\ \cancel{-1}$



$u = 2x$

NOT $\int_0^{\pi/2} \sin(u)\,\frac{du}{2}$!

$$\int_{x=0}^{x=\pi/2} \sin(2x)\,dx = \int_{u=0}^{u=\pi} \sin(u)\,\frac{du}{2} = \frac{1}{2}\left(-\cos u\right)\Big|_{u=0}^{u=\pi}$$

$$= \frac{1}{2}\left(-(-1) - (-1)\right)$$

$$= \underline{\underline{1}}$$

Ex $\int_{\pi/3}^{\pi/2} (\cos 3x)\,e^{(\sin 3x)}\,dx = ?$

$u = \sin 3x$

$du = 3\cos 3x\,dx$

$\frac{du}{3} = \cos 3x\,dx$

$$= \int_{x=\pi/3}^{x=\pi/2} e^u\,\frac{du}{3}$$

$x = \pi/2 \longrightarrow u = \sin\left(\frac{3\pi}{2}\right) = -1$

$x = \pi/3 \longrightarrow u = \sin\left(3 \cdot \frac{\pi}{3}\right) = 0$

$$= \int_0^{-1} \frac{1}{3} e^u\,du$$

$= \frac{1}{3} e^u \Big|_0^{-1} = \underline{\frac{1}{3}\left(e^{-1} - 1\right)}$ | OR: $\frac{1}{3} e^{\sin 3x} \Big|_{x=\pi/3}^{x=\pi/2} = \underline{\frac{1}{3}(e^{-1}-1)}$

Could also do: $u = 3x$, $du = 3\,dx$

$$\int (\cos 3x)\, e^{\sin 3x}\, dx$$

$$= \int (\cos u)\, e^{\sin u} \cdot \frac{du}{3}$$

then take: $t = \sin u$

Ex $\displaystyle\int \frac{x^2 + 16x + 8}{\sqrt{\frac{x}{2} + 1}}\, dx = ?$

could do either:

$u = \frac{x}{2} + 1$

$u = \sqrt{\frac{x}{2} + 1}$

if we take $u = \frac{x}{2} + 1$

then $du = \frac{1}{2} dx$

$2du = dx$

$$\int \frac{x^2 + 16x + 8}{\sqrt{u}}\, 2\, du = 2\int \frac{x^2}{\sqrt{u}} + \frac{16x}{\sqrt{u}} + \frac{8}{\sqrt{u}}\, du$$

and since $u = \frac{x}{2} + 1$, $x = 2u - 2$

so this is

$$= 2\int \frac{(2u-2)^2}{\sqrt{u}} + \frac{16(2u-2)}{\sqrt{u}} + \frac{8}{\sqrt{u}}\, du$$

$$= 2\int \frac{4u^2 - 8u + 4 + 32u - 32 + 8}{\sqrt{u}}\, du$$

$$= 2\int \frac{4u^2 + 24u - 20}{\sqrt{u}}\, du$$

$$= 2\int 4u^{3/2} + 24u^{1/2} - 20u^{-1/2}\, du$$

$= \ldots$

$$= \frac{4}{5}\sqrt{\frac{x}{2}+1}\left(x^2+24x-56\right)+C$$

Q $\displaystyle\int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{dx}{\sqrt{x}+x^{3/2}}$

$\neq \int x^{-1/2} + x^{-3/2}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{2\,du}{1+x}$$

but $u = \sqrt{x}$

$u^2 = x$

$$= \int \frac{2\,du}{1+u^2} = 2\tan^{-1} u + C$$

$$= \underline{\underline{2\tan^{-1}\sqrt{x} + C}}$$