Lecture 5

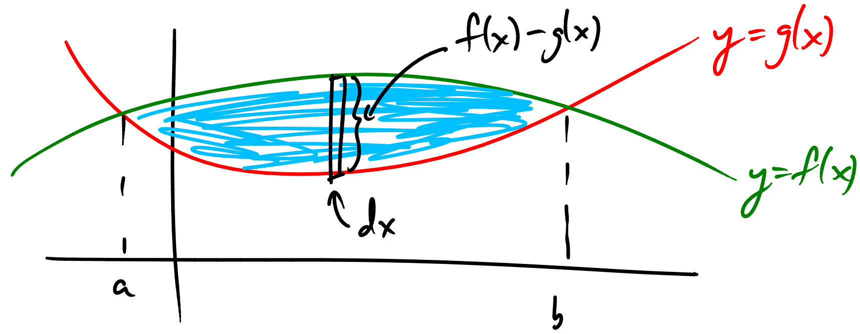
Admin: survey at tinyurl.com/yd9stt3w

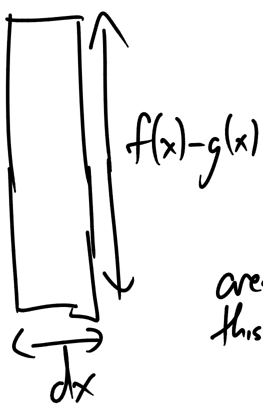
next LM Sat midnight

HW Tue 3am

my office hr Mon 2-3, Thu 5-6 (RLM 9.134)

Last time: Area between curves



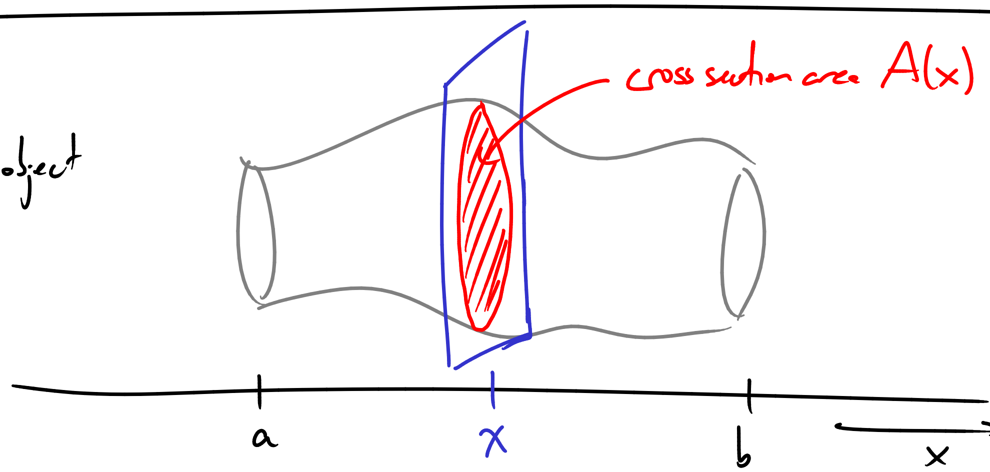


area of this rect $= (f(x)-g(x))\,dx$

$$A = \int_a^b (f(x)-g(x))\,dx$$

Volumes

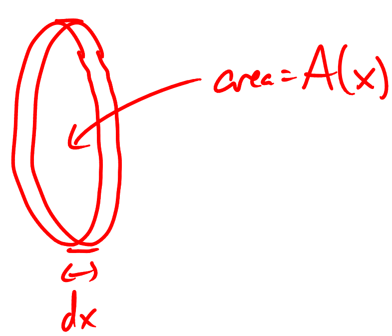
Suppose have some 3-d object



Chop object into thin slices:

volume of slice =

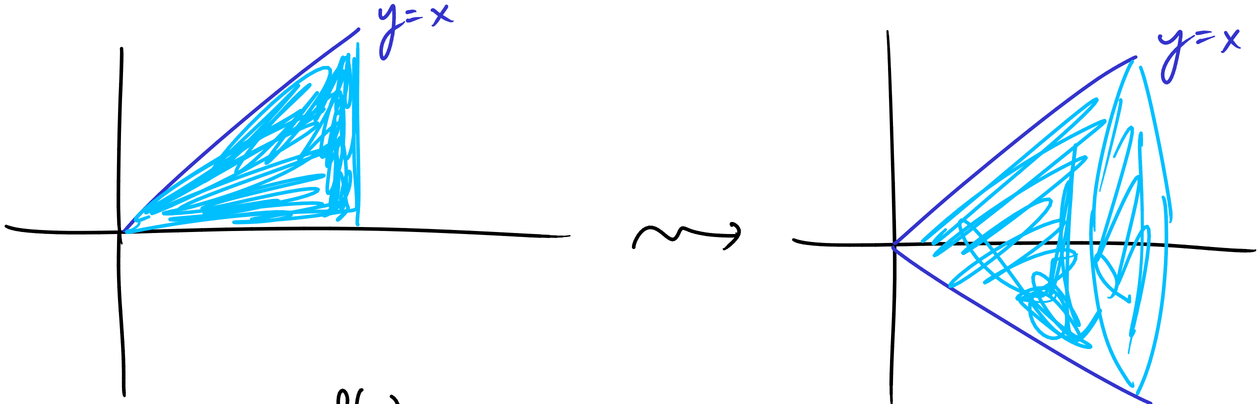
$A(x)\,dx$

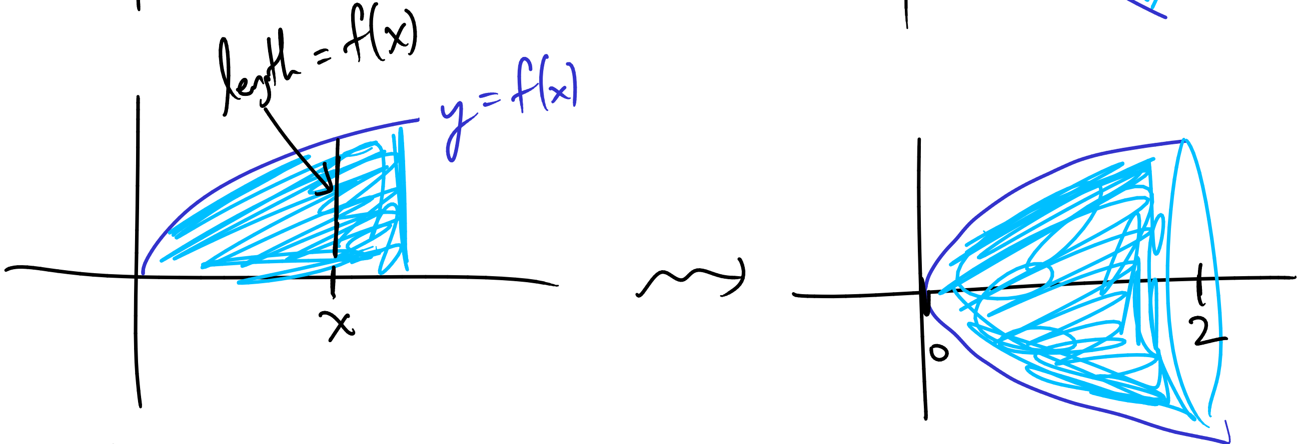


Total volume: $\int_a^b A(x)\,dx$.

One common type of solid: "solid of revolution" — take e.g. the region under some graph and revolve it around, say, the x-axis.

Ex





at fixed x, cross section: circle of radius $f(x)$

Area: $A(x) = \pi f(x)^2$

Q Find the vol. of a solid obtained by rotating the area under $y = \sqrt{x}$ around x-axis, for x from 0 to 2.

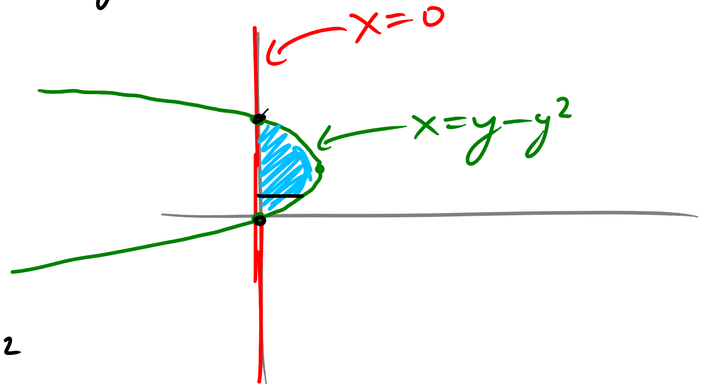
$$V = \int_0^2 A(x)\,dx = \int_0^2 \pi(\sqrt{x})^2\,dx = \int_0^2 \pi x\,dx = \cdots = \underline{\underline{2\pi}}$$

Q Find the vol. of the region obtained by revolving the region between

$x = y - y^2$

$x = 0$

around the y-axis.



cross section at fixed y:

circle of radius $= y - y^2$

$$A(y) = \pi(y-y^2)^2 \qquad V = \int_0^1 \pi(y-y^2)^2\,dy = \dots = \frac{\pi}{30}$$

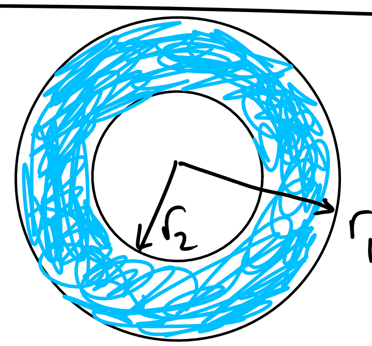
$$= \int_0^1 \pi(y^2-2y^3+y^4)\,dy$$

$$= \pi\left(\frac{1}{3}y^3 - \frac{1}{2}y^4 + \frac{1}{5}y^5\right)\Big|_0^1$$

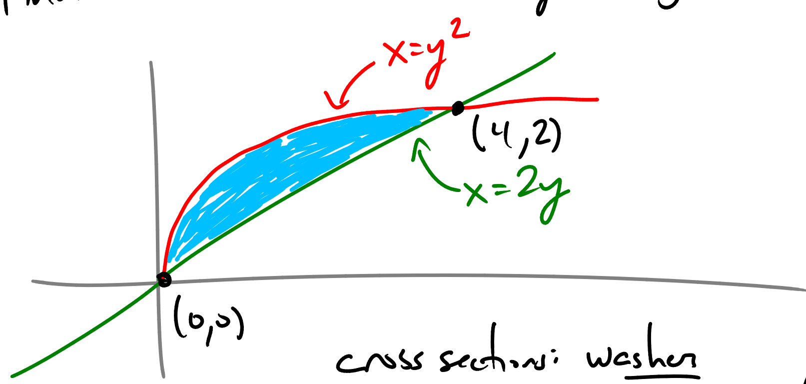
$$= \pi\left(\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right) = \pi\left(\frac{1}{30}\right) = \frac{\pi}{30}$$

Another possibility: cross sections are "washers"

$$A = \pi(r_1^2 - r_2^2)$$

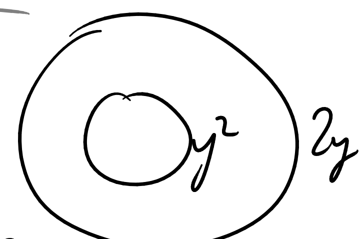


Q: Let R be the region between $y=\sqrt{x}$ and $x=2y$.
Find the vol. of solid obtained by rotating R around the y-axis.



cross sections: washer

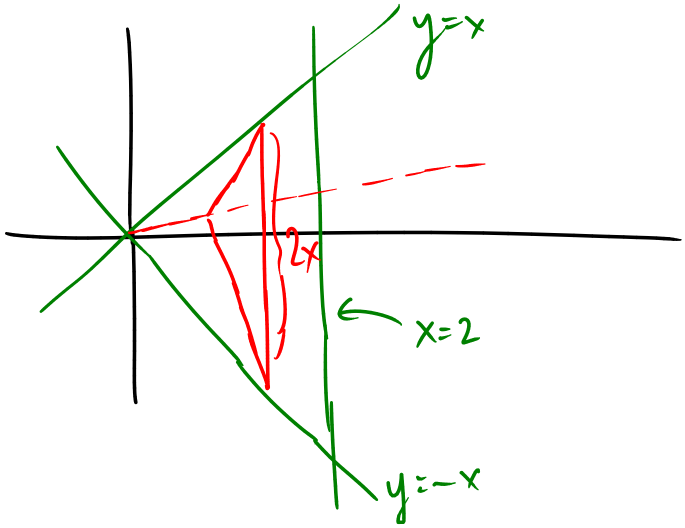
$$A(y) = \pi\left((2y)^2 - (y^2)^2\right)$$



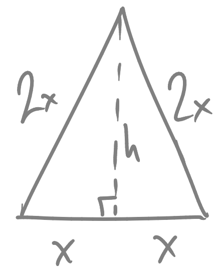
$$V = \int A(y)\,dy = \int_0^2 \pi\left((2y)^2-(y^2)^2\right)$$

$$= \dots = \frac{64\pi}{15}$$

Q Calculate the volume of a solid whose base is the region between $y=x$, $y=-x$ and $x=2$, and whose cross sections at fixed x are equilateral triangles.



$$V = \int A(x)\,dx$$



$$x^2 + h^2 = (2x)^2$$
$$h^2 = 3x^2$$
$$h = \sqrt{3}x$$
$$A = \frac{1}{2}(2x)(\sqrt{3}x) = \sqrt{3}x^2$$

$$A = \int_0^2 \sqrt{3}x^2\,dx = \sqrt{3}\cdot\frac{x^3}{3}\Big|_0^2 = \frac{8\sqrt{3}}{3} \text{ "} \frac{8}{\sqrt{3}}$$

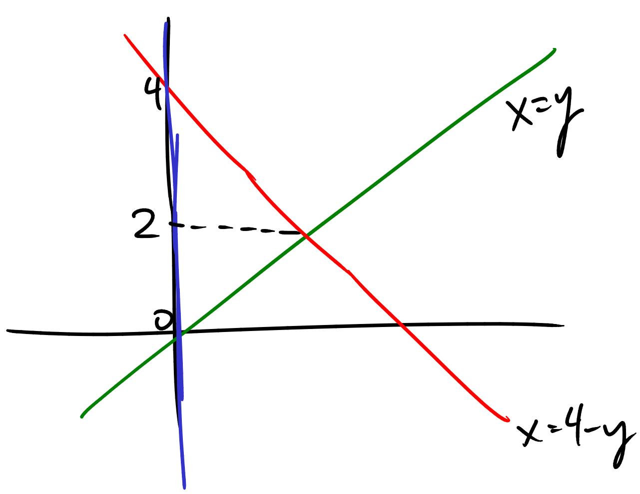
Q Find the volume of solid obtained by revolving the region between

$y = x$

$y = 4 - x$

and the y-axis revolved around y-axis.

Bonus: calculate the volume of a sphere of radius r by slicing it.
$\left(V = \frac{4}{3}\pi r^3\right)$



$$V = \int_0^4 A(y)\,dy$$
$$= \int_0^2 \pi y^2\,dy + \int_2^4 \pi(4-y)^2\,dy$$
$$= \ldots = \frac{8\pi}{3} + \frac{8\pi}{3} = \frac{16\pi}{3}$$