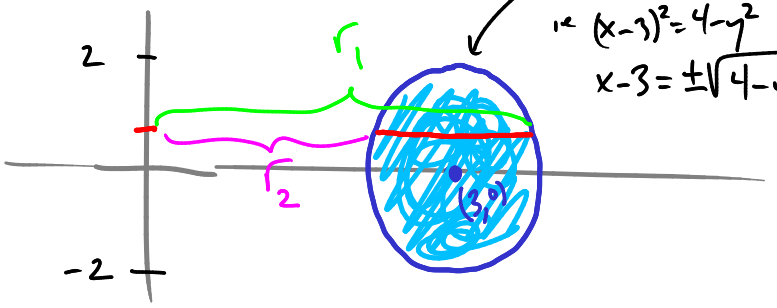
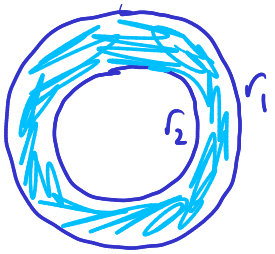
Lecture 6

"Donut" problem from HW:



cross sections at fixed y: washers



$$V=\int_{-2}^{2} A(y)\,dy=\int_{-2}^{2}\pi\left(r_1^2-r_2^2\right)$$

$$r_1=3+\sqrt{4-y^2}$$
$$r_2=3-\sqrt{4-y^2}$$

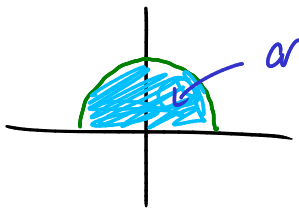
$$=\int_{-2}^{2}\pi\left(\left(3+\sqrt{4-y^2}\right)^2-\left(3-\sqrt{4-y^2}\right)^2\right)dy$$

$$=\int_{-2}^{2}\pi\left(9+6\sqrt{4-y^2}+(4-y^2)\right)-\left(9-6\sqrt{4-y^2}+(4-y^2)\right)dy$$

$$=\int_{-2}^{2}\pi\cdot 12\cdot\sqrt{4-y^2}\,dy$$

what's $\int_{-2}^{2}\sqrt{4-t^2}\,dt$?

$=2\pi$



Integration By Parts

$$\int u\,dv=uv-\int v\,du$$

Q Find $\int x\cos(5x)\,dx$.

try $u=x$ $\quad v=\frac{1}{5}\sin(5x)$

$du=dx$ $\quad dv=\cos(5x)\,dx$

then $\int u\,dv=uv-\int v\,du$

$$=\frac{x}{5}\sin(5x)-\int\frac{1}{5}\sin(5x)\,dx$$

$$=\frac{x}{5}\sin(5x)+\frac{1}{25}\cos(5x)+C$$

Q Find $\int \ln x \, dx$.

write this as $u\,dv$: $u = \ln x$, $v = x$; $du = \frac{dx}{x}$, $dv = dx$

$$uv - \int v\,du$$

$$= (\ln x)\cdot(x) - \int x \cdot \frac{dx}{x}$$

$$= x\ln x - \int 1\,dx = \underline{x\ln x - x + C}$$

Q Find $\int e^t t^2\,dt$.

$u = t^2$, $v = e^t$; $du = 2t\,dt$, $dv = e^t\,dt$

$$uv - \int v\,du$$

$$= t^2e^t - \int e^t(2t\,dt)$$

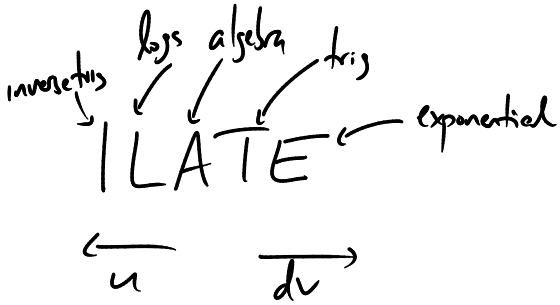
$$= t^2e^t - 2\underbrace{\int e^t t\,dt}_{\int u\,dv}$$

$u = t$, $v = e^t$; $du = dt$, $dv = e^t dt$

$$= t^2e^t - 2(uv - \int v\,du)$$

$$= t^2e^t - 2(te^t - \int e^t dt)$$

$$= \underline{t^2e^t - 2te^t + 2e^t + C}$$



Q $\int_0^{\pi} t\sin(3t)\,dt =$

$u = t$, $v = -\frac{1}{3}\cos(3t)$; $du = dt$, $dv = \sin(3t)\,dt$

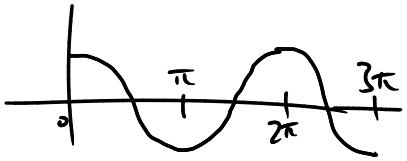
$$uv\Big]_{t=0}^{t=\pi} - \int_{t=0}^{t=\pi} v\,du$$

$$= -\frac{1}{3} t\cos(3t)\Big|_0^{\pi} - \int_0^{\pi} \left(-\frac{1}{3}\cos 3t\right) dt$$

$$= -\frac{\pi}{3}\cos(3\pi) + \frac{1}{3}\int_0^{\pi} \cos 3t\, dt$$

$$= \frac{\pi}{3} + \frac{1}{3}\cdot\frac{1}{3}\sin 3t\Big|_0^{\pi}$$

$$= \frac{\pi}{3}$$



Q $\int e^x \sin x\, dx = ?$

$u = e^x \quad v = -\cos x$

$du = e^x dx \quad dv = \sin x\, dx$

$$\int e^x \sin x\, dx = -e^x\cos x - \int (-\cos x)\, e^x dx$$

$$= -e^x\cos x + \int e^x \cos x\, dx$$

$u = e^x \quad v = \sin x$

$du = e^x dx \quad dv = \cos x\, dx$

$$= -e^x\cos x + \left(e^x \sin x - \int e^x \sin x\, dx\right)$$

So: $2\cdot\int e^x \sin x\, dx = -e^x\cos x + e^x \sin x$

$$\int e^x \sin x\, dx = \frac{1}{2}\left(-e^x\cos x + e^x \sin x\right) + C$$

Q $\int \cos(\sqrt{x})\, dx = ?$

u-sub: $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$dx = 2\sqrt{x}\, du$

$$= \int \cos(u)\,(2\sqrt{x}\, du)$$

$$= \int 2u\cos(u)\, du$$

change names: $u \to t \quad t = \sqrt{x}$

$2\int t\cos(t)\,dt$ do by parts: $u = t \quad v = \sin t$

$du = dt \quad dv = \cos t\,dt$

$= 2(uv - \int v\,du) = 2(t\sin t - \int \sin t\,dt)$

$= 2(t\sin t + \cos t)$

$= 2(\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}) + C$

Q $\int \tan^{-1}(4t)\,dt = ?$