Lecture 9

Exam 1 Tue 7–9pm Jester A121A
bring pencil, ID   no calculator
draft exam has 17 Q's

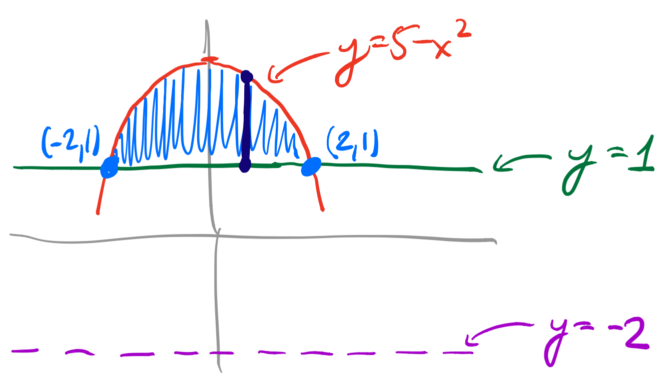
Volumes to find the volume of a solid —
slice vertically or horizontally
then find area of cross section $A(x)$ or $A(y)$

$$V = \int A(x)\,dx \text{ or } \int A(y)\,dy$$

two types:

1) solid of revolution
then cross sections are circles or washers

Ex find vol. of solid we get by revolving the region between $y=1$ and $y=5-x^2$ around the axis $y=-2$.



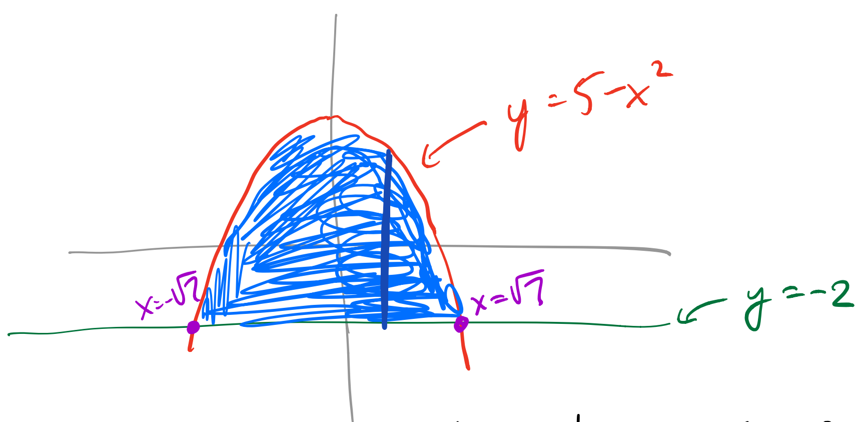
cross sections at fixed x: washer, radii outer $7-x^2$
inner 3

(to get outer radius: distance from $y = 5 - x^2$
to $y = -2$
is $(5-x^2) - (-2) = 7 - x^2$)

So area $A(x) = \pi\left((7-x^2)^2 - 3^2\right)$

$$V = \int_{-2}^{2} A(x)\,dx = \int_{-2}^{2} \pi\left((7-x^2)^2 - 3^2\right)dx$$

An alternative: revolve region between $y = 5 - x^2$
and $y = -2$
around axis $y = -2$.

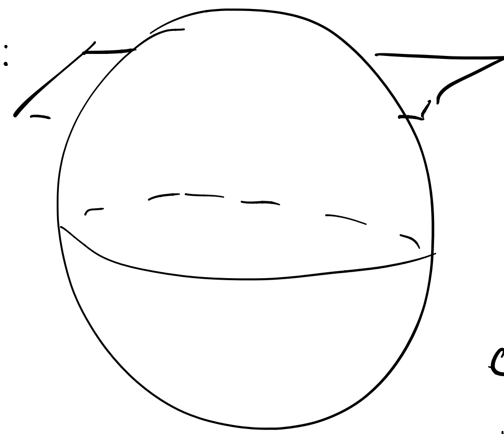


Cross sec. at fixed x: circle radius $r = (5-x^2) - (-2)$
$= 7 - x^2$

$A(x) = \pi(7-x^2)^2$

$$V = \int_{-\sqrt{7}}^{\sqrt{7}} \pi(7-x^2)^2\,dx$$

Volume of sphere:



$x^2+y^2+z^2=1$

cross sec. at fixed z:

$x^2+y^2=1-z^2$

circle in x-y plane

$r=\sqrt{1-z^2}$

$V=\int_{-1}^{1} A(z)\,dz$

$A(z)=\text{area}=\pi r^2=\pi(1-z^2)$

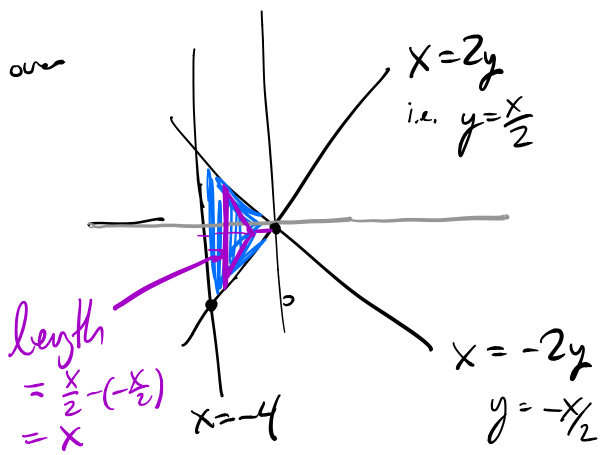
Cross sections are equil. Δ's:

Find vol. of a solid lying over the region between

$x=2y$

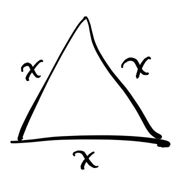
$x=-2y$

$x=-4$

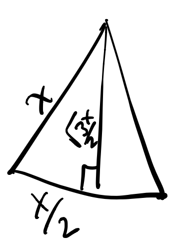


such that slices at fixed x are equil Δ's.

Cross section area:



has area $\frac{\sqrt{3}}{4}x^2$



$\frac{1}{2}bh=\frac{1}{2}(x)\left(\frac{\sqrt{3}x}{2}\right)=\frac{\sqrt{3}}{4}x^2$

$V=\int_{-4}^{0}\frac{\sqrt{3}}{4}x^2\,dx$

IBP: $\int x \cos x \, dx$

$u = x \qquad v = \sin x$

$du = dx \quad dv = \cos x \, dx$

$$\int x \cos x \, dx = uv - \int v \, du$$

$$= x \sin x - \int \sin x \, dx$$

$$= \underline{x \sin x + \cos x + C}$$

$$\int \frac{x e^{-x}}{(x-1)^2} dx$$

$u = x e^{-x} \qquad v = -\frac{1}{x-1}$

$du = (+e^{-x} - x e^{-x}) \, dx \quad dv = \frac{dx}{(x-1)^2}$

$= (1-x) e^{-x}$

$$uv - \int v \, du = -\frac{x}{x-1} e^{-x} - \int -\frac{1}{x-1} (1-x) e^{-x} \, dx$$

$$= -\frac{x}{x-1} e^{-x} - \int e^{-x} \, dx$$

$$= -\frac{x}{x-1}e^{-x} + e^{-x}$$

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a few useful $\int$'s:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x$$

$$\int \sec^2 dx = \tan x$$

$$\int \tan x \sec x \, dx = \sec x$$

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what about $\int \tan x \, dx$?

$$= \int \frac{\sin x}{\cos x}\,dx \qquad u = \cos x$$

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my web site: http://www.ma.utexas.edu/users/neitzke

$$\int_1^2 x^2 f''(x)\,dx \qquad f(1) = 2 \qquad f(2) = 5$$
$$f'(1) = 1 \qquad f'(2) = 7$$

IBP: $u = x^2$ $\quad v = f'(x)$

$du = 2x\,dx$ $\quad dv = f''(x)\,dx$ $\quad \ldots$

$$x^2 f'(x)\Big|_1^2 - \int_1^2 f'(x)\cdot 2x\,dx$$

$\ldots$

FTC I:

$$\frac{d}{dx}\left(\int_5^{x^3} \sin t \, dt\right) = 3x^2 \cdot \sin(x^3)$$

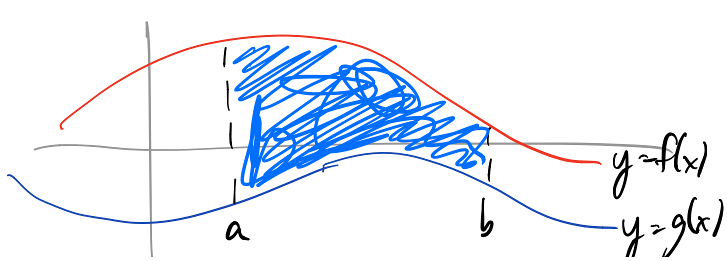
$$\frac{d}{dx}\left(\int_{x^3}^{5} \sin t \, dt\right) = -3x^2 \cdot \sin(x^3)$$

$$\frac{d}{dx}\left(\int_{\ln x}^{x} \sin t \, dt\right) = \sin x - \frac{1}{x}\sin(\ln x)$$

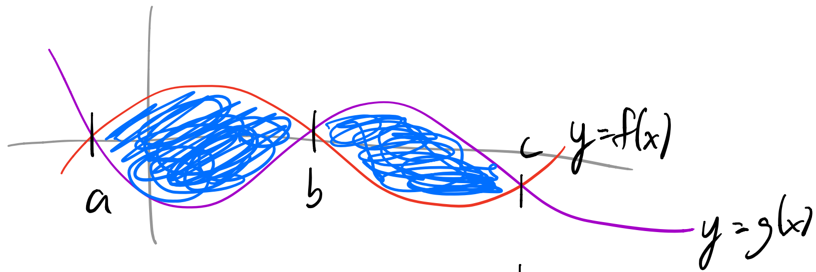
$$\frac{d}{dx}\left(\int_4^7 \sin t \, dt\right) = 0$$

$$\frac{d}{dx}\left(\int_5^{u} \sin t \, dt\right) = \boxed{\frac{du}{dx}}\,\underbrace{\frac{d}{du}\left(\int_5^{u} \sin t \, dt\right)}_{\sin u}$$

$$\frac{d}{dx}\left(\int_{3x^3}^{x^2} \cos t \, dt\right) = 2x\cos(x^2) - 9x^2\cos(3x^3)$$

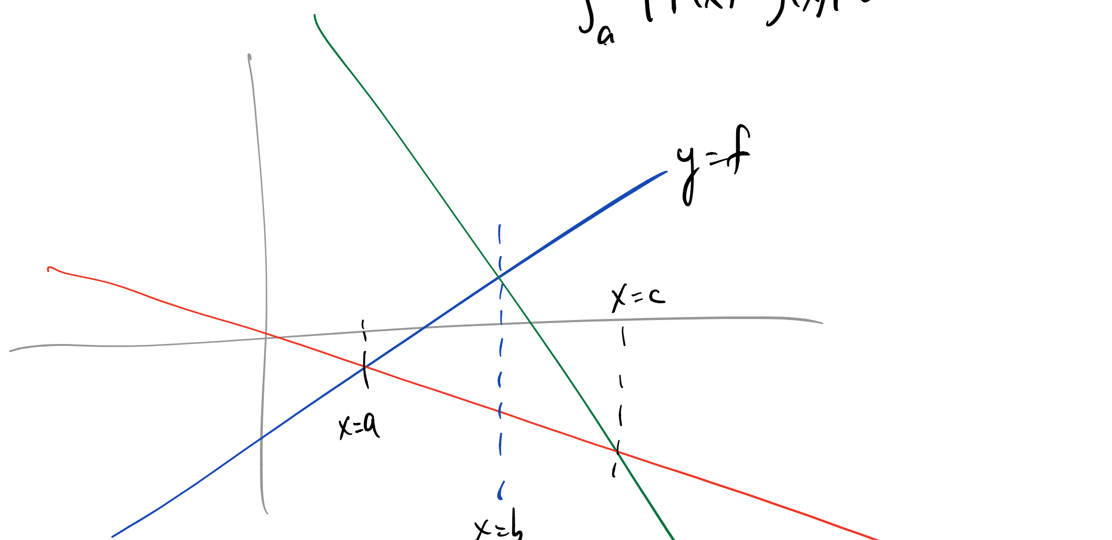


$$\text{area} = \int_a^b \left(f(x) - g(x)\right)\, dx$$



$$\text{area} = \int_a^b f(x) - g(x)\, dx + \int_b^c g(x) - f(x)\, dx$$

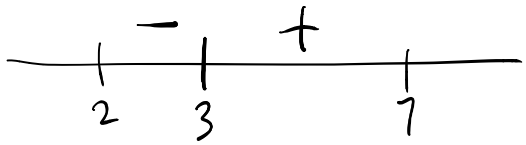
$$= \int_a^c |f(x) - g(x)|\, dx$$



to find area of this triangle:

<u>break it up</u> $\quad A = \int_a^b f(x) - g(x)\, dx + \int_b^c h(x) - g(x)\, dx$

integral of abs value: $\displaystyle\int_2^7 |x-3|\,dx$



on + part: $|x-3| = x-3$

on − part: $|x-3| = 3-x$

so $\displaystyle\int_2^7 |x-3|\,dx = \int_2^3 3-x\,dx + \int_3^7 x-3\,dx$

$\displaystyle\int \sin x \cdot e^x\,dx$ $\quad u = \sin x \quad v = e^x$

$du = \cos x\,dx \quad dv = e^x\,dx$

$= \sin x\, e^x - \int v\,du$

$= \sin x\, e^x - \int e^x \cos x\,dx$ $\quad u = \cos x \quad v = e^x$

$du = -\sin x\,dx \quad dv = e^x\,dx$

$= \sin x\, e^x - \left(\cos x\, e^x - \int e^x(-\sin x)\,dx \right)$

$= \sin x\, e^x - \cos x\, e^x - \int e^x \sin x\,dx$

$\longrightarrow 2\int \sin x\, e^x\,dx = \sin x\, e^x - \cos x\, e^x$

$\displaystyle\int \sin x\, e^x\,dx = \frac{1}{2}\left(\sin x\, e^x - \cos x\, e^x\right)$