Lecture 9

Exam 1 Tue 7–9pm Jester A121A

bring pencils, ID no calculators

draft exam has 17 Q's

covers start of course thru IBP.

HW1,2,3 and the IBP part of HW4

Sec 5.3–7.1

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x$$

$$\int \sec x \tan x\,dx = \sec x \qquad \int \sec^2 x\,dx = \tan x$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$= \int -\frac{du}{u} = -\ln|u| = -\ln|\cos x|$$

$$= \ln|\sec x|$$

$$\int \sec x\,dx = \ln|\sec x + \tan x|$$

$$\int \sec x\,dx = \int \frac{\sec^2 x + \tan x \sec x}{\sec x + \tan x}\,dx$$

$u = \sec x + \tan x \quad du = (\sec^2 x + \sec x \tan x)\,dx$

$= \ln|u| = \ln|\sec x + \tan x|$

$$\int_1^2 (1+\ln x)\, x^x \, dx \qquad x^x = \left(e^{\ln x}\right)^x = e^{x \ln x}$$

$$= \int_{x=1}^{x=2} (1+\ln x)\, e^{x\ln x}\, dx \qquad u = x\ln x$$

$$du = (1+\ln x)\, dx$$

$$= \int_{u=0}^{u=2\ln 2} e^u \, du$$

$$= e^u \Big|_0^{2\ln 2}$$

$$e^{\ln x} = x$$

$e^{2\ln 2} = \left(e^{\ln 2}\right)^2 = 2^2 = 4$

$$= e^{(2\ln 2)} - e^0 \qquad \ln(e^x) = x \qquad \ln(1) = 0$$

$$= 4 - 1 = 3$$

Volumes

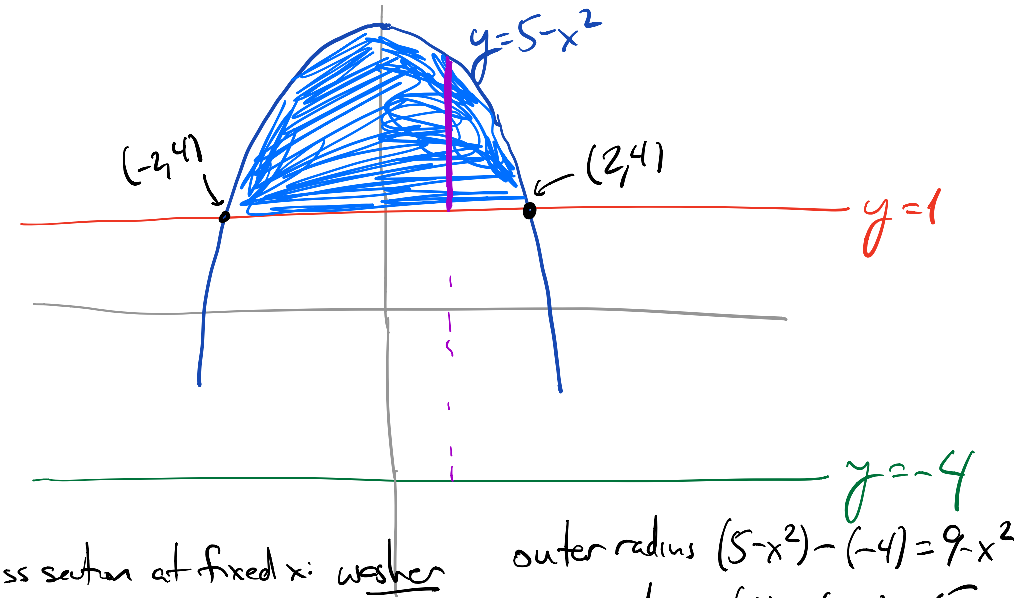
To find volume of some solid: take **slices** at constant x or at constant y

find the area of each slice

$A(x)$ or $A(y)$

then $V = \int A(x)\, dx$ or $\int A(y)\, dy$

Ex Find vol. of the solid we get by revolving the region between $y = 1$ and $y = 5 - x^2$ around axis $y = -4$.



Cross section at fixed x: washer

outer radius $(5-x^2)-(-4)=9-x^2$

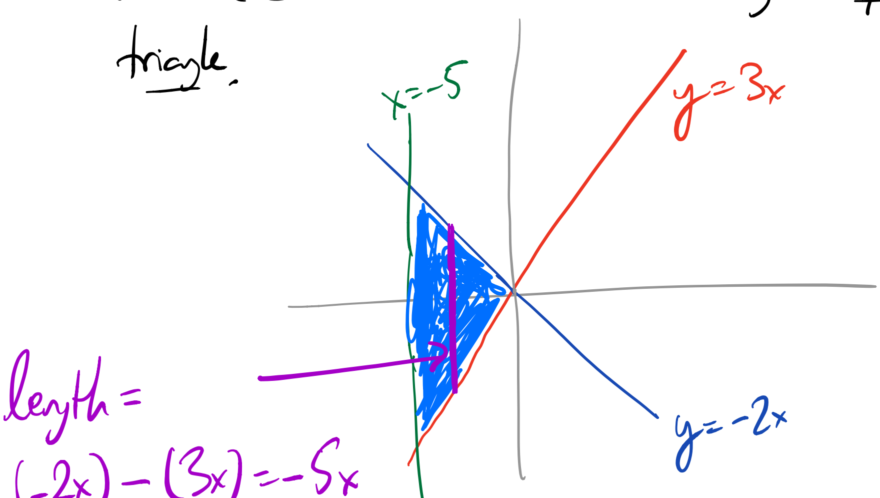
inner radius $(1)-(-4)=5$

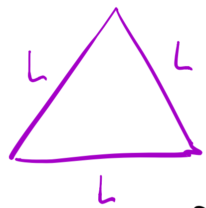
$$A(x)=\pi\left((9-x^2)^2-5^2\right)$$

$$V=\int_{-2}^{2}\pi\left((9-x^2)^2-5^2\right)\,dx$$

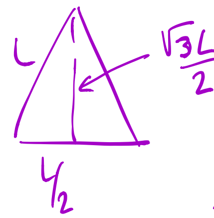
Ex Find volume of solid lying over a base which is the region between $y=3x$ $\quad y=-2x$ $\quad x=-5$

with the cross section at fixed x being an equilateral triangle.





$A = \frac{\sqrt{3}}{4} L^2$



$$A = \frac{1}{2} bh = \frac{1}{2} L \cdot \frac{\sqrt{3}L}{2} = \frac{\sqrt{3}}{4} L^2$$

$$V = \int_{-5}^{0} A(x)\, dx$$

$$= \int_{-5}^{0} \frac{\sqrt{3}}{4} (-5x)^2\, dx = \ldots$$

$$= \int_{-5}^{0} \frac{\sqrt{3}}{4} (25x^2)\, dx$$

$$= \frac{\sqrt{3}}{4} \cdot 25 \left. \frac{x^3}{3} \right|_{-5}^{0}$$

$$= \frac{\sqrt{3}}{4} \cdot 25 \cdot \frac{1}{3} (0^3 - (-5)^3) = \ldots$$

IBP:

$$\int e^x \sin x\, dx$$

$u = \sin x \quad v = e^x$

$du = \cos x\, dx \quad dv = e^x dx$

$$\int e^x \sin x\, dx = \sin x\, e^x - \int e^x \cos x\, dx$$

$u = \cos x \quad v = e^x$

$du = -\sin x\, dx \quad dv = e^x dx$

$$= \sin x\, e^x - \left(\cos x\, e^x - \int e^x (-\sin x\, dx) \right)$$

$$= \sin x\, e^x - \cos x\, e^x - \int e^x \sin x\, dx$$

$$\rightarrow 2 \int e^x \sin x\, dx = \sin x\, e^x - \cos x\, e^x$$

$$\int e^x \sin x \, dx = \frac{1}{2}(\sin x \, e^x - \cos x \, e^x) + C$$

FTC 1:

$$\frac{d}{dx}\left(\int_{-7}^{5\ln x} \tan(t)\, dt\right) = \frac{5}{x}\tan(5\ln x)$$

$$\frac{d}{dx}\left(\int_{5\ln x}^{-7} \tan(t)\, dt\right) = -\frac{5}{x}\tan(5\ln x)$$

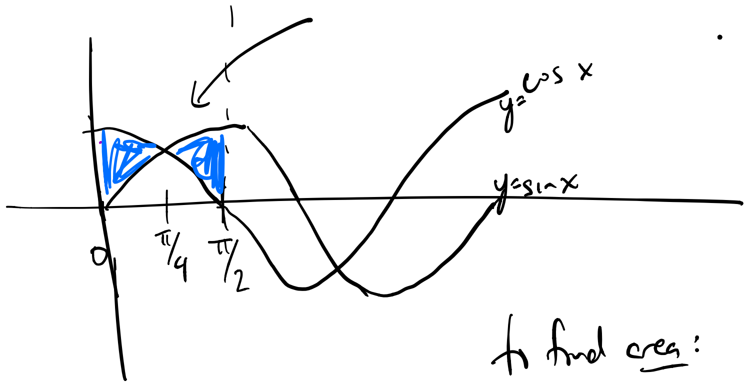
$$\frac{d}{dx}\left(\int_{x^6}^{3x^2} \ln(t)\, dt\right) = 6x\ln(3x^2) - 6x^5\ln(x^6)$$

$$= 6x\ln(3x^2) - 36x^5\ln(x)$$

$$\int \frac{d}{dx}(\sin x \ln x \tan^{-1} x \, e^x)\, dx$$

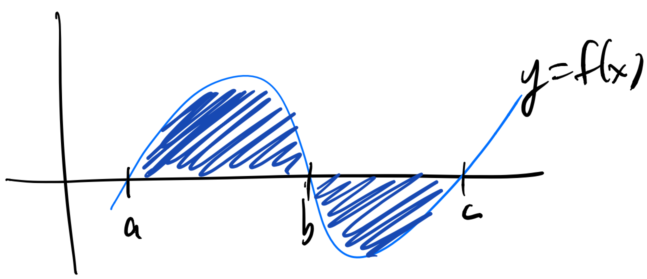
$$= \sin x \ln x \tan^{-1} x \, e^x + C$$

$$\int_a^b \frac{d}{dx}F(x)\, dx = F(b) - F(a)$$



to find area:

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

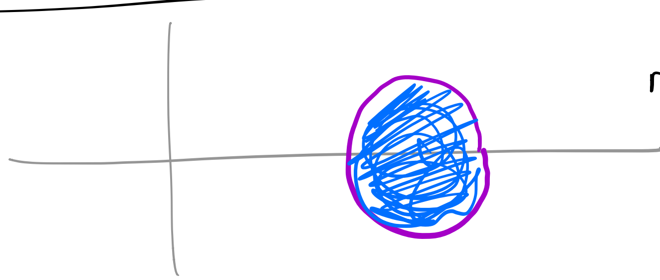


"find the area between y = f(x) and the x-axis as x goes from a to c"

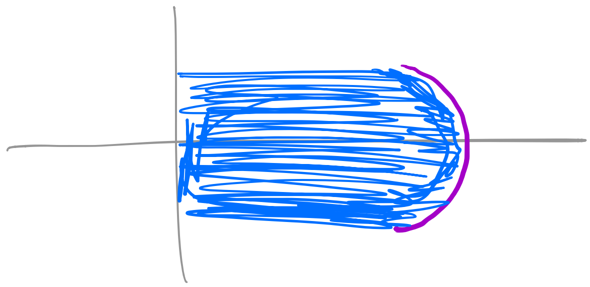
"area" is always positive!

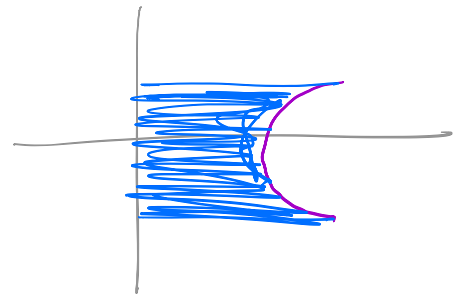
means $\int_a^b f(x)\,dx + \int_b^c -f(x)\,dx$

or $\int_a^c |f(x)|\,dx$



revolve around y-axis and find vol.





---

substitution:

$$\int \frac{3\cos x - \sin x}{3\sin x + \cos x}\,dx$$

$u = 3\sin x + \cos x$

$du = (3\cos x - \sin x)\,dx$

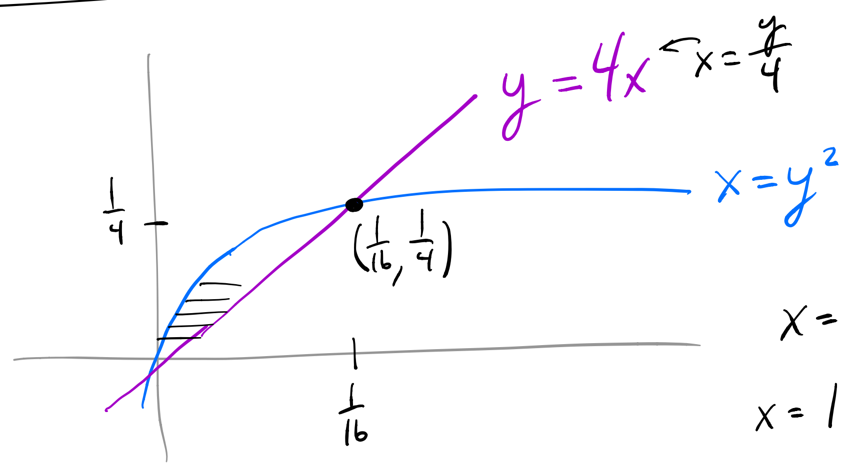
$$\int \frac{du}{u}$$

$$= \ln|u| = \ln|3\sin x + \cos x| + C$$

$$\int \cos x\, e^{\sin x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$



$x = (4x)^2$

$x = 16x^2$

$x = \frac{1}{16}$

$$A = \int_0^{\frac{1}{4}} \left(\frac{y}{4} - y^2\right) dy$$

or: revolve around y-axis

$$V = \pi \int^{\frac{1}{4}} \left(\frac{y}{4}\right)^2 - (y^2)^2\, dy$$

$$\int \frac{x+7}{\sqrt{x+3}}\,dx \qquad u = x+3$$

$$u-3 = x$$

$$\int \frac{u+4}{\sqrt{u}}\,du \qquad du = dx$$

$$= \int u^{1/2} + 4u^{-1/2}\,du = \cdots$$

---

completing the square: $\displaystyle\int \frac{1}{x^2+4x+5}\,dx \qquad (x+2)^2 = x^2+4x+4$

$$= \int \frac{1}{(x+2)^2+1}\,dx \qquad u = x+2$$

$$= \tan^{-1}(x+2) + C$$

$$x^2 + ax + b = \left(x + \frac{a}{2}\right)^2 + \left(b - \frac{a^2}{4}\right)$$

$$\left(x + \frac{a}{2}\right)^2 = x^2 + ax + \frac{1}{4}a^2$$