Lecture 15

Exam 2 Tue Oct 31 7–9pm Jester A121A

Covers Sec 7.2 – 14.3 i.e. everything from trig integrals $\longrightarrow$ partial derivatives

second part of HW4 $\longrightarrow$ first part of HW8

Last time: partial derivatives

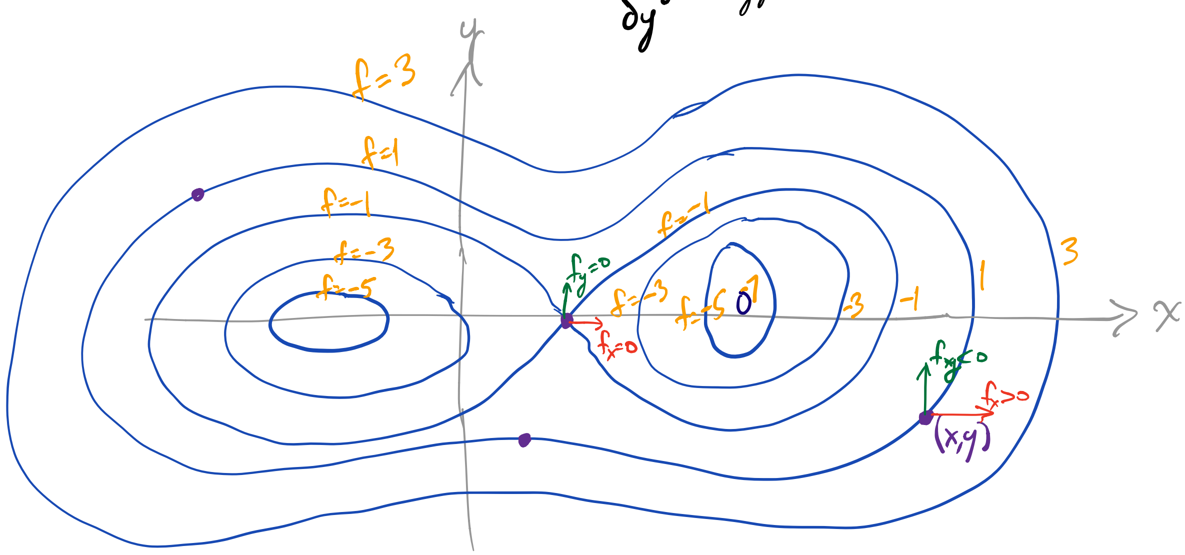
$$f(x,y) \rightsquigarrow \frac{\partial f}{\partial x} = f_x \rightsquigarrow \frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial f}{\partial x} = f_x \rightsquigarrow \frac{\partial^2 f}{\partial y \partial x} = f_{yx}$$

$$f(x,y) \rightsquigarrow \frac{\partial f}{\partial y} = f_y \rightsquigarrow \frac{\partial^2 f}{\partial x \partial y} = f_{xy}$$

$$\frac{\partial f}{\partial y} = f_y \rightsquigarrow \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

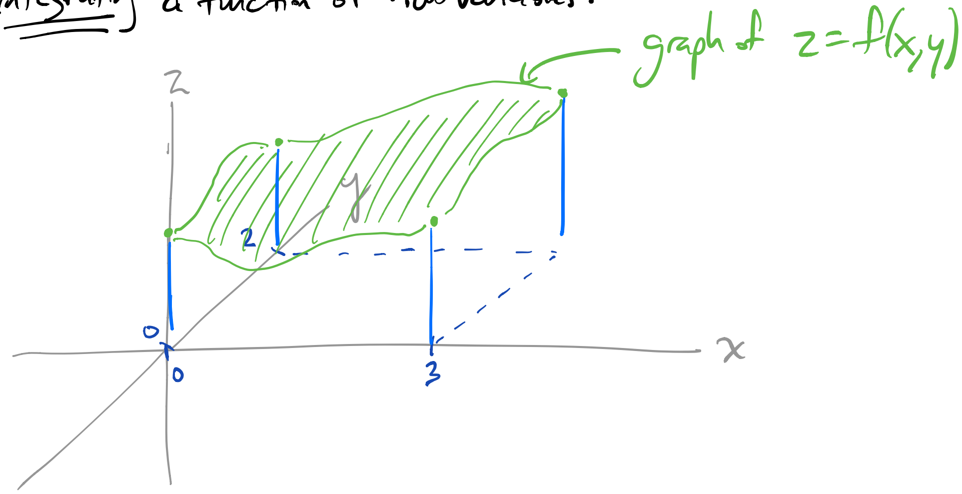
these two are equal!



Q At (x,y) is f_x positive, negative or zero? $f_x > 0$

f_y positive, negative or zero? $f_y < 0$

How about *integrating* a function of two variables?



Q: What is the total volume under this graph

(ie volume between graph $z=f(x,y)$ and the xy-plane $z=0$) ?

Cut by planes at fixed y: $V=\int_0^2 A(y)\,dy$

$$A(y)=\int_0^3 f(x,y)\,dx \quad \text{gives cross section area}$$

So $V=\int_0^2\left[\int_0^3 f(x,y)\,dx\right]dy$

Q Suppose $f(x,y)=4xy+3x^2$ What's V?

$$V=\int_0^2\left[\int_0^3 4xy+3x^2\,dx\right]dy$$

$$=\int_0^2\left[2x^2y+x^3\Big|_{x=0}^{x=3}\right]dy$$

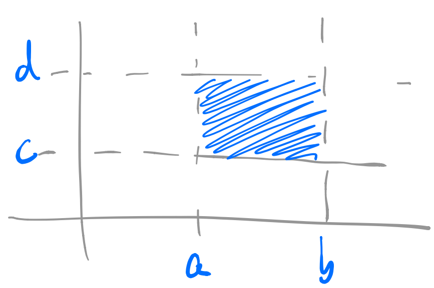
$$=\int_0^2\left[18y+27-0\right]dy$$

$$= 9y^2 + 27y \Big|_0^2$$
$$= 36 + 54$$
$$= \underline{\underline{90}}$$

__Rk__ We __could__ also do it in the other order:

$$\int_0^3 \left[\int_0^2 4xy + 3x^2 \, dy \right] dx$$
$$= \int_0^3 \left[2xy^2 + 3x^2 y \Big|_{y=0}^{y=2} \right] dx$$
$$= \int_0^3 8x + 6x^2 \, dx$$
$$= 4x^2 + 2x^3 \Big|_0^3$$
$$= 36 + 54 = \underline{\underline{90}}$$

Either order of integration gives __same__ answer ("Fubini's Thm")



$$\int_a^b \left[\int_c^d f(x,y) \, dy \right] dx$$
$$= \int_c^d \left[\int_a^b f(x,y) \, dx \right] dy$$

We also write this as $\displaystyle\iint_R f(x,y) \, dA$

$dy \uparrow \boxed{dA}$ $\overrightarrow{dx}$

$dA = dx\,dy$

$R = \{ a \le x \le b, \ c \le y \le d \}$

Q. If $R = \{1 \le x \le 2,\ 0 \le y \le \pi\}$

and $f(x,y) = y\sin(xy)$ what is $\iint_R f(x,y)\, dA$?

$$\int_0^{\pi}\left(\int_1^2 y\sin(xy)\cdot dx\right) dy$$

$$= \int_0^{\pi}\left(-y\frac{\cos(xy)}{y} \Big|_{x=1}^{x=2} \right) dy$$

$$= \int_0^{\pi} -\cos(2y) + \cos(y)\ dy$$

$$= -\frac{\sin(2y)}{2} + \sin(y) \Big|_0^{\pi}$$

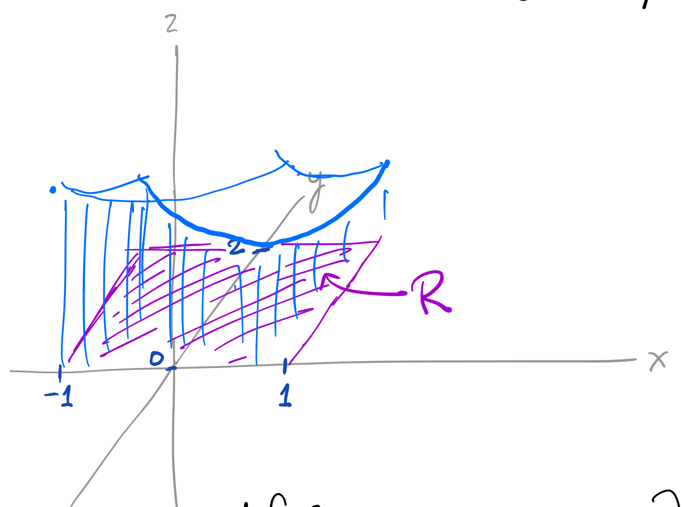
$$= \underline{\underline{0}}\ !$$

[OR: could do $\int_1^2 \left(\underbrace{\int_0^{\pi} y\sin(xy)\ dy}_{\text{do by IBP, this is harder}} \right) dx$]

Q Find the volume of the solid which lies under the graph of

$$z = f(x,y) = 4 + x^2 - y^2$$

and over the rectangle $R = \begin{cases} -1 \le x \le 1 \\ 0 \le y \le 2 \end{cases}$.



$f(x,y) \geq 0$
whenever $(x,y) \in R$.

$$\text{So, } V = \int_{-1}^{1} \left[\int_0^2 4 + x^2 - y^2 \, dy \right] dx$$

$$= \int_{-1}^{1} \left(4y + x^2 y - \tfrac{1}{3} y^3 \Big|_{y=0}^{y=2} \right) dx$$

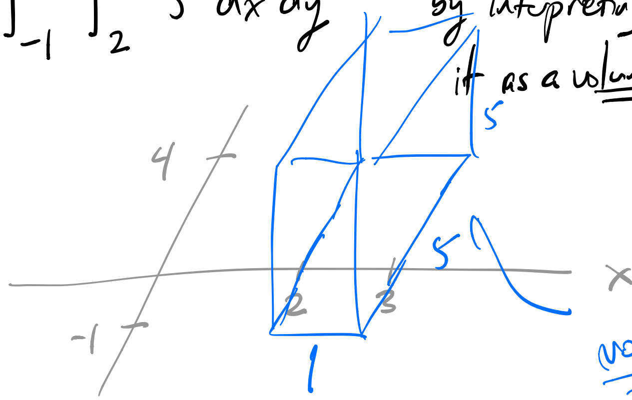
$$= \int_{-1}^{1} \left(8 + 2x^2 - \tfrac{8}{3} \right) dx$$

$$= \int_{-1}^{1} \tfrac{16}{3} + 2x^2 \, dx$$

$$= \frac{16x}{3} + \frac{2}{3}x^3 \Big|_{-1}^{1}$$

$$= \left(\tfrac{16}{3} + \tfrac{2}{3} \right) - \left(-\tfrac{16}{3} - \tfrac{2}{3} \right) = \frac{36}{3} = \underline{\underline{12}}$$

Q Find $\int_{-1}^{4}\int_{2}^{3} 5 \, dx \, dy$ by interpreting it as a volume.



vol = 25