Lecture 16

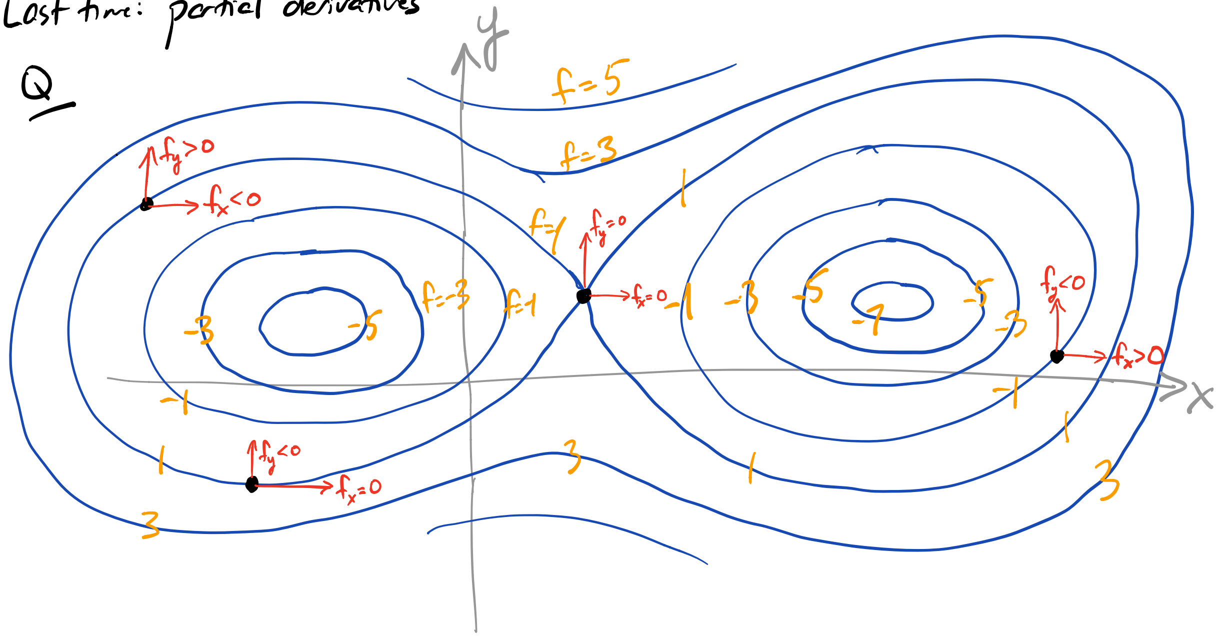
Exam 2 Tue Oct 31 Jester A121A

covers Sec 7.2 - 14.3 i.e. trig integrals ⟶ partial derivatives

second ½ of HW04 ⟶ first ½ of HW08

Last time: partial derivatives

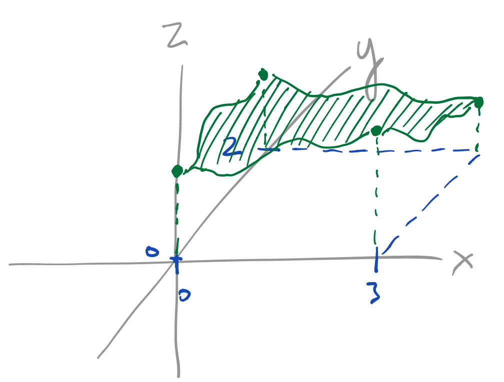
Q



Q At the point (x,y) is f_x positive, negative or zero?

is f_y pos, neg or zero?

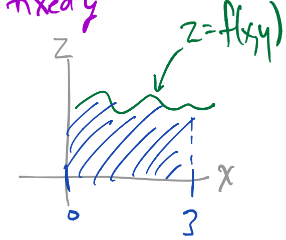
Integrating functions of two variables $\qquad f=f(x,y)$



Q: What is the total volume under this graph

$$\left(\begin{array}{ll} \text{i.e. the volume between the graph of} & z = f(x,y) \\ \text{and the xy-plane} & z = 0 \end{array}\right)?$$

Cut by planes at fixed y: $V = \int_0^2 A(y)\,dy$ ← cross section area at fixed y



$$A(y) = \int_0^3 f(x,y)\,dx$$

So: $$V = \int_0^2 \left(\int_0^3 f(x,y)\,dx \right) dy$$

Q Suppose $f(x,y) = 4xy + 3x^2$. Find the volume of the solid lying over the rectangle $0 \le x \le 3$, $0 \le y \le 2$ and under the graph $z = f(x,y)$.

$$V = \int_0^2 \left(\int_0^3 4xy + 3x^2\,dx \right) dy$$

$$= \int_0^2 \left(2x^2y + x^3 \Big|_{x=0}^{x=3} \right) dy$$

$$= \int_0^2 (18y + 27 - 0)\,dy$$

$$= 9y^2 + 27y \Big|_0^2$$

$$= 36 + 54 = \underline{\underline{90}}$$

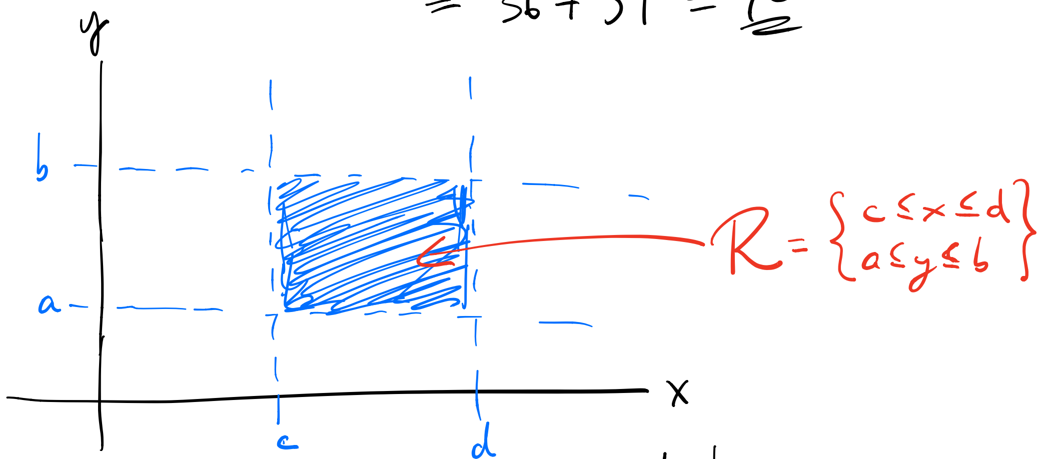
Could also do: $$V = \int_0^3 \left(\int_0^2 4xy + 3x^2\,dy \right) dx$$

$$= \int_0^3 \left(2xy^2 + 3x^2y \Big|_{y=0}^{2}\right) dx$$

$$= \int_0^3 8x + 6x^2 \, dx$$

$$= 4x^2 + 2x^3 \Big|_0^3$$

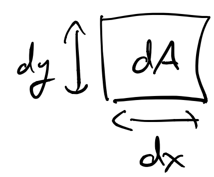
$$= 36 + 54 = \underline{\underline{90}}$$



$$\int_a^b \left(\int_c^d f(x,y)\, dx\right) dy = \int_c^d \left(\int_a^b f(x,y)\, dy\right) dx$$

also call this $\iint_R f(x,y)\, dA$

$dA = dx\,dy = dy\,dx$



$\underline{Q}$ If $R = \{1 \leq x \leq 2,\ 0 \leq y \leq \pi\}$ and $f(x,y) = y\sin(xy)$

what is $\iint_R f(x,y)\, dA$?

It is $\displaystyle\int_0^{\pi}\left[\int_1^2 y\sin(xy)\,dx\right]dy$

$\left(\text{or: } \displaystyle\int_1^2\left[\int_0^{\pi} y\sin(xy)\,dy\right]dx,\ \text{but that's harder}\right)$

Now, $\displaystyle\int_1^2 y\sin(xy)\,dx = y\cdot\int_1^2 \sin(xy)\,dx$

$$= y\cdot\left(-\frac{1}{y}\cos(xy)\Big|_{x=1}^{x=2}\right)$$

$$= -\left(\cos(2y)-\cos(y)\right)$$

$$= \cos(y)-\cos(2y)$$

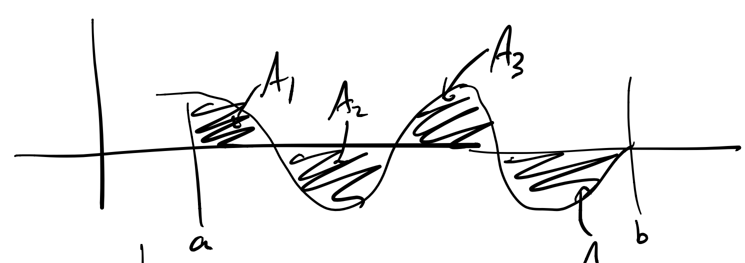
Then, $\displaystyle\int_0^{\pi}\left[\int_1^2 y\sin xy\,dx\right]dy$

$$= \int_0^{\pi} \cos(y)-\cos(2y)\,dy$$

$$= \sin(y)-\frac{1}{2}\sin(2y)\Big|_0^{\pi}$$

$$= \underline{\underline{0}}.$$

How can it be 0? Remember 1-d $\int$ counts areas with signs



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3 - A_4$$

could be 0.

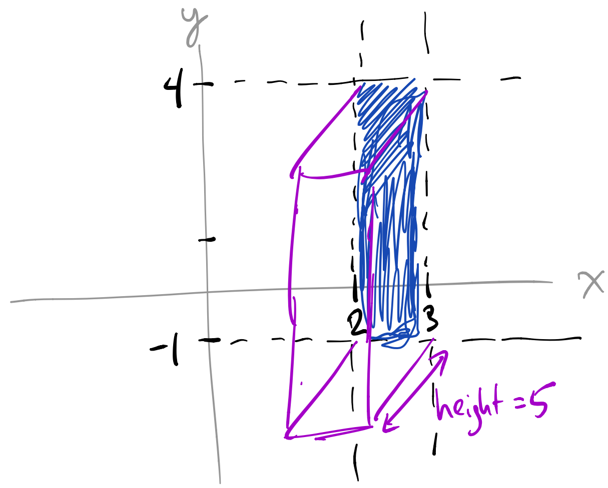
2-d $\iint_R f(x,y)\, dA$ counts volume with signs

the part where $f(x,y) > 0$ gives volume

——— " — $f(x,y) < 0$ gives minus volume

they can cancel each other!

Q Find $\int_{-1}^{4}\int_{2}^{3} 5\, dx\, dy$ by interpreting it as a volume.



box with dimensions

$1 \times 5 \times 5$

volume $= \underline{\underline{25}}$

Q Find the volume of the solid which lies under the graph of

$$z = f(x,y) = 4 + x^2 - y^2$$

and over the rectangle $-1 \le x \le 1$

$0 \le y \le 2$.

(NB: for all (x,y) in this rectangle $f(x,y) \ge 0$.)

"nota bene"

the volume is $V = \iint_R f(x,y)\, dA$

$$= \iint_R 4 + x^2 - y^2 \, dA$$

$$= \int_{-1}^{1} \left[\int_0^2 4 + x^2 - y^2 \, dy \right] dx$$

$$= \int_{-1}^{1} \left(4y + x^2 y - \frac{1}{3} y^3 \Big|_{y=0}^{2} \right) dx$$

$$= \int_{-1}^{1} 8 + 2x^2 - \frac{8}{3} \, dx$$

$$= \int_{-1}^{1} \frac{16}{3} + 2x^2 \, dx$$

$$= \frac{16}{3} x + \frac{2}{3} x^3 \Big|_{-1}^{1}$$

$$= 6 + 6 = \underline{\underline{12}}$$