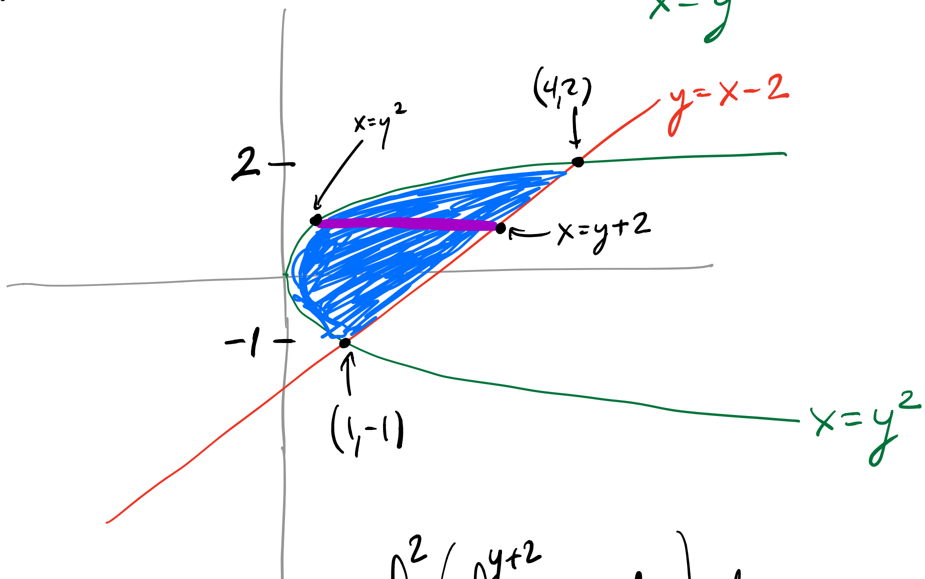
Last time: double integrals over "general regions"

Q $\displaystyle\iint_D y\, dA$ $\qquad$ D bounded by $y = x-2$, $x = y^2$



$$\int_{-1}^{2}\left(\int_{y^2}^{y+2} y \, dx\right) dy$$

Inside integral: $\displaystyle\int_{y^2}^{y+2} y\, dx = xy \Big|_{y^2}^{y+2} = (y+2)y - (y^2)y$
$\qquad = y^2 + 2y - y^3$

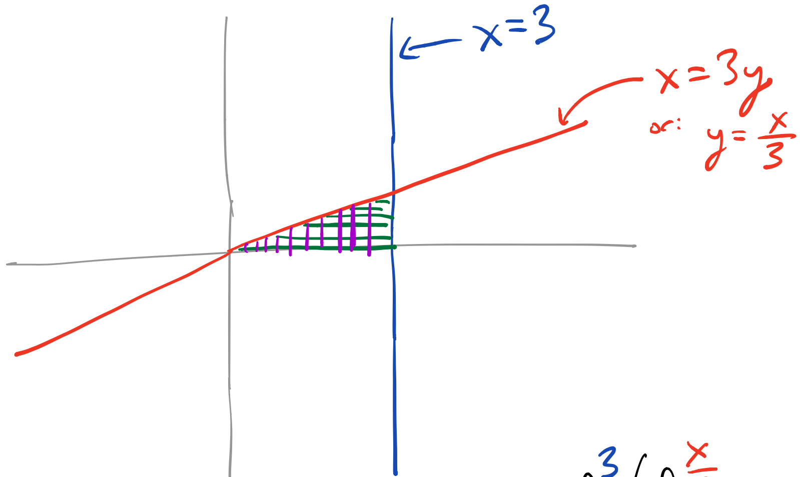
outside integral: $\displaystyle\int_{-1}^{2} y^2 + 2y - y^3\, dy = \frac{1}{3}y^3 + y^2 - \frac{1}{4}y^4 \Big|_{-1}^{2}$
$\qquad = \left(\frac{8}{3} + 4 - 4\right) - \left(-\frac{1}{3} + 1 - \frac{1}{4}\right)$
$\qquad = \frac{8}{3} + \frac{1}{3} + \frac{1}{4} - 1 = \underline{\underline{\frac{9}{4}}}$

$$\left[e^{(x^2)} \neq (e^x)^2 = e^{2x}\right]$$

Q $\displaystyle\int_0^1 \int_{3y}^{3} e^{x^2}\,dx\,dy = ?$

$\boxed{\textbf{Q} \displaystyle\int_0^1 \int_{\sqrt{x}}^{1} \sqrt{y^3+1}\,dy\,dx = ?}$

Can't just do the inside integral directly.



Try doing it instead by vertical slices: $\displaystyle\int_0^3 \left(\int_0^{\frac{x}{3}} e^{x^2}\,dy \right) dx$

inside int. $\displaystyle\int_0^{x/3} e^{x^2}\,dy = y e^{x^2} \Big|_{y=0}^{y=x/3}$

$$= \frac{x}{3} e^{x^2}$$

outside int: $\displaystyle\int_0^3 \frac{x}{3} e^{x^2}\,dx$

$u = x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$$= \int_0^9 \frac{du}{2\cdot 3} e^u$$

$$= \frac{1}{6}\left(e^9 - 1\right)$$

Exam 2 Tue Oct 31 Jester A121A. 7–9pm

Sections 7.2, 7.3, 7.4, 7.5, 7.8, 14.3

7.2: trig $\int$ · 7.3: trig sub · 7.4: partial frac · 7.5: "strategy for integration" · 7.8: improper int · 14.3: partial deriv.

Draft exam has 16 problems

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rules for convergence/divergence:

$$(c>0) \quad \int_c^\infty \frac{1}{x^p}\,dx \text{ is } \begin{cases} \text{convergent if } p>1 \\ \text{divergent if } p\le 1 \end{cases}$$

$$\int_a^c \frac{1}{(x-a)^p}\,dx \text{ is } \begin{cases} \text{convergent if } p<1 \\ \text{divergent if } p\ge 1 \end{cases}$$

Ex $\int_4^\infty \frac{1}{\sqrt{x}}\,dx$ is divergent $(p=\frac{1}{2}<1)$

$\int_7^\infty \frac{1070}{\sqrt{x}}\,dx$ also divergent

$\int_5^{10} \frac{1}{x-5}\,dx$ divergent $(p=1)$

$2\int_6^7 \frac{1}{x-3}\,dx$ is not improper $\left(=2\ln\frac{4}{3}\right)$ $\left(=\ln\frac{16}{9}\right)$

Partial fractions

$$\int \frac{1}{x^3+4x^2}\,dx$$

$x^3+4x = x(x^2+4)$

$\rightarrow \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$x^3+4x^2 = x^2(x+4)$

$$\frac{1}{x^3+4x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$$

mult by $x^3+4x^2 \longrightarrow$

$\quad\quad\;\; \shortparallel$

$\quad\;\; x^2(x+4)$

$$1 = Ax(x+4) + B(x+4) + Cx^2$$

$$1 = Ax^2 + 4Ax + Bx + 4B + Cx^2$$

$$1 = (A+C)x^2 + (4A+B)x + 4B$$

$A+C=0$

$4A+B=0$

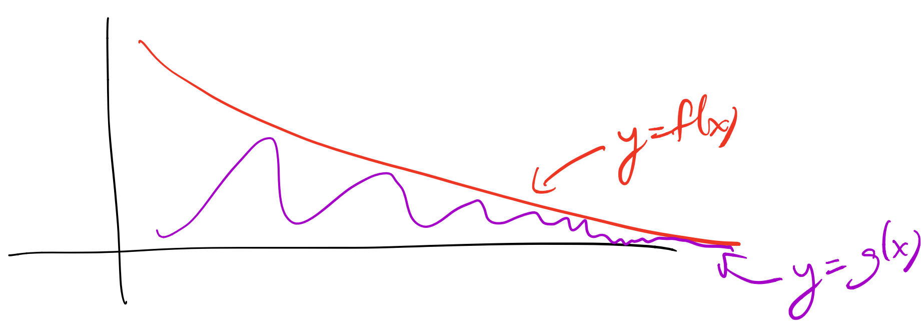
$4B=1 \longrightarrow B = \frac{1}{4} \quad A = -\frac{1}{16}$

$C = \frac{1}{16}$

Comparison Theorem:

If $f(x) \geq 0$ and $\int_c^\infty f(x)\,dx$ converges

and $f(x) \geq g(x) \geq 0$ then $\int_c^\infty g(x)\,dx$ also converges



e.g. $\displaystyle\int_5^\infty \frac{1}{x^3+1}\,dx$ converges, because

$$\frac{1}{x^3+1} < \frac{1}{x^3}$$

and $\displaystyle\int_5^\infty \frac{1}{x^3}\,dx$ converges

Q $\displaystyle\int \frac{1}{\sqrt{4x-x^2}}\,dx = ?$

complete the square

$$4x-x^2 = -(x^2-4x)$$

$(x-2)^2 = x^2-4x+4$

$$= -\left((x-2)^2-4\right)$$

$$= 4-(x-2)^2$$

$$= \int \frac{1}{\sqrt{4-(x-2)^2}}\,dx$$

$x-2 = 2\sin\theta$

$\vdots$

or $x-2 = 2u$

$dx = 2du$

$$\int \frac{1}{\sqrt{4-4u^2}}\,2du$$

$$= \int \frac{du}{\sqrt{1-u^2}}$$

$= \sin^{-1}(u) =$

$\sin^{-1}\left(\frac{x-2}{2}\right)$

$$\int \tan^{-1} x \, dx$$

integrate by parts: $u = \tan^{-1} x$ $\quad v = x$

$du = \dfrac{dx}{1+x^2}$ $\quad dv = dx$

$$= x \tan^{-1} x - \underbrace{\int \frac{x}{1+x^2} \, dx}$$

do by u-sub $u = 1+x^2$

or just write $\frac{1}{2} \ln |1+x^2|$