

NOAH KRAVITZ'S SENIOR ESSAY PROJECT (FALL 2019)

Essentials:

- Project: Lonely Runner Conjecture.
- Adviser: Stefan Steinerberger (now <https://faculty.washington.edu/steinerb/>).
- Written product: “Barely lonely runners and very lonely runners”, preprint available at <https://arxiv.org/abs/1912.06034v1>.

Description:

- The Lonely Runner Conjecture is a hard, well-known, and (at the present) unsolved problem in Diophantine analysis. The Wikipedia page for the problem (https://en.wikipedia.org/wiki/Lonely_runner_conjecture) has a nice description. I decided to write an original research thesis instead of an expository thesis. I knew from the outset that it was unrealistic to hope to resolve the entire problem, so I was on the lookout for partial results and special cases where improvements were possible.
- Within the first few weeks of the semester, I stumbled upon a question that I ended up focusing on for most of the rest of the semester. The question was essentially about whether or not “near-equality cases” for the Lonely Runner Conjecture must be uniformly bounded away from “true equality cases” (details in paper). In particular, I formulated a “sharpened” Lonely Runner Conjecture that describes how near-equality cases cannot get too close to true equality cases.
- For the first half of the semester, I worked on confirming my sharpened conjecture when the number of runners is small. Once I had a better feel for the problem, I turned to descriptions of more general phenomena for larger numbers of runners. Near the end of the semester, I returned to the existing literature and found a few previous results that I could incorporate into my new framework.

Work-flow:

- I did some background reading but spent more time thinking about new ideas.
- I met with Professor Steinerberger weekly to discuss progress, and we exchanged emails as things came up.
- I kept an informal write-up of my results as I went, and I started assembling the formal paper over Thanksgiving Break. (I wish I had started the formal writing process earlier.)