Review of Computations in Calculus

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Disclaimer

- These slides are designed exclusively for students attending section 1, 2 and 3 for the course 640:244 in Fall 2013. The author is not responsible for consequences of other usages.
- These slides may suffer from errors. Please use them with your own discretion.

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- Note: DO NOT forget to take absolute values in the natural logarithm.

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- How to compute: Use integration by parts to solve the special case that $a = e$, then again use $\log_a x = \ln x / \ln a$.

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- Antiderivative: Not interesting at least in 244. So forget it.

Hyperbolic trigonometric functions

- Definitions:

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The rest two are left as exercises for technique of substitution.

More formulas

$$\int \frac{1}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

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- How to compute: Either by trigonometric substitution or by breaking rational functions.

More formulas

$$\int \frac{1}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

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$$\int \frac{1}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$$

- How to compute: Either by trigonometric substitution or by hyperbolic substitution.

More detail about the last integral

Example:

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$$x = a \sinh t = \frac{e^t - e^{-t}}{2} \Rightarrow 2\frac{x}{a} e^t = e^{2t} - 1$$

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$$= \int \frac{1}{a \cosh t} a \cosh t dt = \int dt = t + C$$

$$x = a \sinh t = \frac{e^t - e^{-t}}{2} \Rightarrow 2\frac{x}{a}e^t = e^{2t} - 1 \Rightarrow e^{2t} - 2\frac{x}{a}e^t - 1 = 0$$

More detail about the last integral

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$$\int \frac{1}{\sqrt{x^2+a^2}}dx = t + C = \ln\left|\frac{x \pm \sqrt{x^2+a^2}}{a}\right| + C$$

The End