QUIZ 6 FOR CALC 4

(1) (4 pt) Find the general solution of the equation

$$y''(t) + 2y'(t) + y(t) = te^t$$

Solution: This is just a standard problem, provided you don't use the wrong template. The correct way you to do this is to let

$$P(t) = e^t(At + B)$$

And to compute

$$P'(t) = Ae^t(t + 1) + Be^t = (At + A + B)e^t,$$

$$P''(t) = Ae^t(t + 1) + (A + B)e^t = (At + 2A + B)e^t.$$

So

$$P'' + 2P' + P = (At + 2A + B + 2At + 2A + 2B + At + B)e^t = (4At + 4A + 4B)e^t = te^t$$

In order to make an equality, it is required simultaneously that

$$4A = 1, 4A + 4B = 0$$

This tells you $A = 1/4, B = -1/4$. So a particular solution is

$$P(t) = \frac{1}{4}e^t(t - 1)$$

Remind that you are asked for the general solution! So the final answer should be

$$y(t) = c_1e^{-t} + c_2te^{-t} + \frac{1}{4}e^t(t - 1).$$

If you used a wrong template, for example, $P(t) = Ate^t$, legally there is nothing wrong if the A DOES SATISFY THE REQUIREMENTS. Unfortunately you won't be able to find such an A, because you won't be able to eliminate the e^t term. In more detail:

$$P(t) = Ate^t, P'(t) = A(t + 1)e^t, P''(t) = A(t + 2)e^t,$$

$$P'' + 2P' + P = 4Ate^t + 4Ae^t = te^t.$$

Therefore it is required SIMULTANEOUSLY that

$$4A = 1, 4A = 0$$

You won't be able to find such an A

(2) (4 pt) Find the general solution of the equation

$$y''(t) + y(t) = \cos t$$

I have already leaved the blackboard for reference. The template you should be using is TOTALLY THE SAME as the homework problem. So basically speaking, you don't even have to perform the computation yourself, just copy them from the blackboard.

Taking into account of the performance, I finally have to add in the details:

If you know that

$$c_1 \sin t + c_2 \cos t$$

is the general solution of the homogeneous equation, then it is immediate that the first try fails (think about why). Now we perform the second try. Let

$$P(t) = t(A \sin t + B \cos t).$$

Then

$$\begin{aligned} P'(t) &= A(1 \cdot \sin t + t \cdot \cos t) + B(1 \cdot \cos t + t \cdot (-\sin t)) \\ &= (A \sin t + t \cos t) + B(\cos t - t \sin t) \\ P''(t) &= A(\cos t + (\cos t - t \sin t)) + B(-\sin t - (\sin t + t \cos t)) \\ &= A(2 \cos t - t \sin t) + B(-2 \sin t - t \cos t) \end{aligned}$$

Therefore

$$P + P'' = 2A \cos t - 2B \sin t = \cos t$$

So

$$2A = 1, B = 0,$$

and therefore

$$P(t) = \frac{1}{2} t \sin t.$$

So the general solution is

$$y(t) = c_1 \sin t + c_2 \cos t + \frac{1}{2} t \sin t.$$