QUIZ 8 FOR CALC 4

Name:______________________________________ **RUID:**__________________________

(1) (4 pt) Find the general solution of

$$y'''(t) - 2y''(t) + y'(t) - 2y(t) = e^{2t}$$

The first step is to factorize the characteristic polynomial.

$$u^3 - 2u^2 + u - 2 = u^2(u-2) + (u-2) = (u^2+1)(u-2)$$

Many student spent a lot of time on this step, using the not very good looking long division algorithm. It is nice if you can perform the long division. But it is not always necessary to perform the unpleasant substraction. At least in this example you should have observed that $u-2$ is a common factor. In general performing long division requires you to guess a potential factor. This is probably the most unpleasant procedure that is not always possible. Let me illustrate an another example: how to factorize

$$x^4 + x^2 + 1.$$

This polynomial, as a function, is always strictly positive. So you won't be able to guess a root because there is no root. So how can you guess? The right way to proceed is to look for a square. Notice that

$$\begin{aligned} x^4 + x^2 + 1 &= x^4 + 2x^2 + 1 - x^2 = (x^2+1)^2 - x^2 \\ &\ \text{(Aha! Difference of squares)} \\ &= (x^2 + 1 + x)(x^2 - x + 1) = (x^2 + x + 1)(x^2 - x + 1) \end{aligned}$$

I hope this example will liberate you from the wrong impression that long division can solve everything. At least it cannot from this example. There are various tricks to factorize a polynomial. I'll see if I can find some good references.

Then you can conclude that $2, \pm i$ are the roots of the characteristic polynomial. So the general solutionis

$$y(t) = c_1 e^{2t} + c_2 \sin t + c_3 \cos t$$

BUT JUST FOR THE HOMOGENEOUS PART! To find the general solution of this equation, you still need to find a particular solution. Please consult Dr. Z's solution to Attendance Quiz 16 for steps and please, make sure you understand everything! The final answer to the problem should be

$$y(t) = c_1 e^{2t} + c_2 \sin t + c_3 \cos t + \frac{1}{5} t e^t$$

(2) (2 pt) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{bmatrix}.$$

Find out $A \cdot B$.

Please remember the rule that <u>the number at (i, j)-spot of $A \cdot B$</u> is always **the product i-th row vector of A with the j-th column vector of B**. Many students got this correct but there are students confused on the $(1, 2)$-spot and $(2, 1)$-spot. The computation is as follows:

$$A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 6 + 2 \times 5 + 3 \times 4 & 1 \times 3 + 2 \times 2 + 3 \times 1 \\ 4 \times 6 + 5 \times 5 + 6 \times 4 & 4 \times 3 + 5 \times 2 + 6 \times 1 \end{bmatrix} = \begin{bmatrix} 28 & 10 \\ 73 & 28 \end{bmatrix}.$$

(3) (2 pt) What is the general solution of $ay''(t) + by'(t) + cy(t) = 0$ if the equation $au^2 + bu + c = 0$ has a multiple root r?

What I expect you to give is just simply

$$y(t) = e^{rt}(c_1 + c_2 t).$$

It would be nice that you can notice $r = -b/2a$, although I did not really need that.