Challenging Problem for Calc4 (Mar. 26, 2014)

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Let $f(r) = a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0$ be a polynomial and let D be the differential operator d/dt. Use the exponential shift law to prove the following fact: if α is a root of the polynomial equation $f(r) = 0$ of multiplicity s, then for the following ODE

$$f(D)y = a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = e^{\alpha t} p_m(t)$$

where $p_m(t)$ is a polynomial, the template

$$P(t) = t^k e^{\alpha t}(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_1 t + A_0)$$

fails for all $k < s$. And the template will succeed when $k = s$.