Lecture 11 on June 11

0. Reduction of Order

0.1. Variation of Parameter

0.2. Application to the first order differential equation

0.3. Application to second order homogeneous ODE

Examples: $y'' - 2ry' + r^2 y = 0, y_1 = e^{rt}$, $ty'' - y' - 4t^3 y = 0, y_1 = \sin t^2$.

1. Free Vibration

1.1. Physics background and equation of motion: A mass is attached with a spring vertically. Let u be the displacement of the mass FROM THE EQUILIBRIUM (which characterizes how the mass is stretched "further" from where it stays still). Then the equation of motion in general looks like

$$mu'' + \gamma u' + ku = 0$$

where m stands for the mass, γ represent the damping, k is the spring constant.

1.2. Undamped Case: Natural Frequency, Amplitude and Phase

Example: Suppose that a mass weighing 10 lb stretches a spring 2 in. If the mass is displaced an additional 2 in and is then set in motion with an initial upward velocity of 1 ft/s, determine the position of the mass at any later time. Also determine the period, amplitude, and phase of the motion.

1.3. Underdamped Case: $\gamma < \sqrt{4km}$. Quasi-frequency, Quasi-Period.

Example: A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is given an additional 6 in displacement in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/s. Formulate the IVP and solve it.

1.4. Overdamped Case: $\gamma > \sqrt{4km}$

Example: A mass weighing 4 lb stretches a spring 1.5 in. The mass is in a medium that exerts a viscous resistance of 15 lb when the mass has a velocity of 3 ft/s. Suppose the mass is given an additional 6 in displacement. Find out the condition for the velocity such that the mass passes through the equilibrium (overshoot).

Added June 15th: In class I actually used the following data: A mass weighing 4lb stretches a spring 2 in.

1.5. Critically Damped Case: $\gamma = \sqrt{4km}$

Example: A mass weighing 4 lb stretches a spring 1.5 in. Suppose that the mass is given an additional 6 in displacement in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 12 lb when the mass has a velocity of 3 ft/s. Formulate the IVP and solve it