Math 244
Summer 2015
Hour Exam 2
6/25/14
Time Limit: 80 Minutes

Name (Print): ____________________

This exam contains 9 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	10	
3	10	
4	10	
5	15	
6	15	
7	25	
Total:	100	

1. For the ODE

$$y'' + 7y' + 12y = 0$$

(a) (5 points) Find the general solution.

(b) (5 points) With the parameterized initial value $y(0) = 1, y'(0) = \alpha$, determine the critical value of α when the behavior of the solution changes.

(c) (5 points) Note that the initial value $y(0)$ is specified as a positive number, if the solution is eventually negative this means its graph passes the x-axis at some time. In this case, find this time and express it in terms of α.

2. For the ODE

$$x^2y'' - 3xy' + 4y = 0, x > 0$$

(a) (5 points) Find the general solution.

(b) (5 points) With the initial value $y(1) = 1, y'(1) = 3$, solve the IVP.

3. For the following second order ODE

$$y'' - y' - 2y = e^t + e^{-t}$$

(a) (5 points) Find the complementary solution

(b) (5 points) Find a particular solution to formulate the general solution.

4. For the ODE

$$y^{(4)} + 2y^{(3)} + 4y'' + 8y' = 2t + 2 + 3\cos t + 6\sin t$$

(a) (5 points) Find the complementary solution

(b) (5 points) Find a particular solution and then formulate the general solution.

5. A mass of 2 kg is attached with a spring with the spring constant being 8 N/m. Suppose there is no damping. At $t = 0$ the mass is stretched further 0.1 meter down from the equilibrium and then released.

   (a) (5 points) Find the displacement of the mass from the equilibrium at any later time t.

   (b) (2 points) Find the natural frequency of the vibration.

   (c) (3 points) Suppose in addition there is an external force $2 \sin \omega t$. Find ω such that resonance happens.

   (d) (5 points) Find the displacement of the mass from the equilibrium at any later time t under the condition of the force.

6. A mass of 2 kg is attached with a spring with the spring constant being 8 N/m. Suppose there is a damping force with the damping coefficient being γ N·s/m. At $t = 0$ the mass is stretched further 0.1 meter down from the equilibrium and then released.

   (a) (5 points) Determine the conditions on γ for the vibration to be respectively underdamped, critically-damped and overdamped.

   (b) (5 points) In case the vibration is critically damped, find the displacement of the mass from the equilibrium and determine if it will pass through the equilibrium.

   (c) (5 points) Suppose in addition there is an external force $2 \sin 2t$. With the same initial conditions, find the transient solutions and the steady-state solutions.

7. For the ODE

$$x^2y'' - (x+2)xy' + (x+2)y = x^3e^x$$

(a) (5 points) Verify that $y_1 = x$ solves the homogeneous ODE

$$x^2y'' - (x+2)xy' + (x+2)y = 0$$

(b) (10 points) Find the complementary solution using the technique of reduction of orders (i.e. variation of parameters).
Hint: You should set $y_2(t) = u(t)y_1(t)$ then find u by solving $y_1u'' + (2y_1' + py_1)u' = 0$

Note: The problem continues at the next page

(c) (10 points) Find a particular solution using the method of variation of parameter.

Hint: The formula you should use is $y = y_1 \int \frac{-y_2 g}{W(y_1, y_2)} dt + y_2 \int \frac{y_1 g}{W(y_1, y_2)} dt$

Hint: You may have to use integration by parts.