MATH 310 FALL 2019
PROBLEM SET 8

Due Thursday, Nov 14th, at 11:35 AM, in class.

(1) Show that
$$\int_{-\infty}^{\infty} \frac{\cos x}{(1+x^2)^2} dx = \frac{\pi}{e}$$

(2) Show that
$$\int_{-\infty}^{\infty} \frac{\sin^2 x}{1+x^2} dx = \frac{\pi}{2}\left[1 - \frac{1}{e^2}\right]$$

(3) Evaluate, indicating the range of values for a, b.
$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)} dx$$

(4) Show that for $a > 1$ real parameter
$$\int_{0}^{\pi} \frac{\sin^2 \theta}{a + \cos\theta} d\theta = \pi\left[a - \sqrt{a^2-1}\right]$$

(5) Show that for real parameter $0 \leq r < 1$
$$\int_{-\pi}^{\pi} \frac{1-r^2}{1-2r\cos\theta + r^2} \frac{d\theta}{2\pi} = 1$$

(6) By integrating around the boundary of a pie-slice domain of angle $2\pi/b$ ($b > 1$ real parameter), show that
$$\int_{0}^{\infty} \frac{dx}{1+x^b} = \frac{\pi}{b\sin(\pi/b)}$$

(7) Show that for $0 < a < 1$ real parameter
$$\int_{0}^{\infty} \frac{x^{-a}}{1+x} dx = \frac{\pi}{\sin(\pi a)}$$

(8) Show that for $0 < a, b$ real parameters ($a \neq b$) (Hint: Complexify $\frac{(\log x)^2}{(x+a)(x+b)}$)
$$\int_{0}^{\infty} \frac{\log x}{(x+a)(x+b)} dx = \frac{(\log a)^2 - (\log b)^2}{2(a-b)}$$

Le calcul des residus constitue la source naturelle des integrates defines (E. LINDELÖF)