QUALIFYING EXAM SYLLABUS

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Algebraic Topology

• Fundamental group, Van Kampen’s theorem, Covering Spaces.

• Homology: Singular and cellular homology, excision, long exact sequence of a pair, Mayer-Vietoris sequence. Universal Coefficient theorem. Brouwer fixed point theorem, hairy ball theorem, Borsuk-Ulam theorem.

• Cohomology: Singular and cellular cohomology, cup/cap products. Universal Coefficient theorem, Kunneth formula, Poincare duality, Thom isomorphism.


• Characteristic Classes: Stiefel-Whitney, Euler, Chern.

References: A. Hatcher, Algebraic Topology (Chapters 1, 2, 3.1-3.3, 4.1-4.3) Milnor and Stasheff, Characteristic Classes.

Riemannian Geometry

• Differentiable manifolds, Riemannian Metrics.

• Affine and Riemannian connections.

• Geodesics: Geodesic flow, minimizing properties of geodesics, convex neighborhoods.

• Curvature: Tensor curvature, sectional and Ricci curvature.

• Jacobi fields: Jacobi equation, conjugate points, second variation formula.

• Second fundamental form and the fundamental equations.

• Complete manifolds: Hopf-Rinow and Hadamard theorems.
• Spaces of constant curvature.
• Curvature and Topology: Rauch comparison theorem. Classical results: Bonnet-Myers, Synge-Weinstein, Preissman.

References: M. Do Carmo Riemannian Geometry
P. Petersen Riemannian Geometry
J. Lee Riemannian Manifolds: An Introduction to Curvature

Partial Differential Equations (Minor)
• Four important Linear PDE: Transport, Laplace’s, Heat and Wave equations.
• Theory of Distributions: Distribution, properties. Convolution, Fourier transform.
• Sobolev Spaces: Definitions, Approximation, Extensions, Traces. Sobolev inequalities, Compactness, Poincare’s inequality.
• Second-order Parabolic and Hyperbolic equations: Definitions, Existence, Regularity, Maximum principle.
• Calculus of Variations: First and second variation, existence of minimizers. Regularity and constraints.
• Hamilton-Jacobi Equations: Viscosity solutions, uniqueness.

References: L. C. Evans Partial Differential Equations (Chapters 2, 3.1-3.3, 5, 6, 7.1, 7.2, 8.1-8.5, 10.1, 10.2)
L. Hormander The Analysis of Linear Partial differential Operators I