

Hyperbolic Limits of Cantor sets S^c in S^3

(j. w / Tommaso Cremaschi)

Souto-Stover (2013), Cremaschi-Souto (2018)

Hyperbolizable Cantor sets

Cremaschi (2018) Class of hyperbolizable

M , exhaustion $\{M_i\}$, ∂M_i incompressible components w/ bounded genus.

Here, $M_i \subseteq M$ is T_1 -injective

Flexibility of examples \rightarrow Knot complements

Purcell-Sato (2010) $M \hookrightarrow \mathbb{S}^3$

one ended hyperbolic manifold of

finite type $\Rightarrow M$ is the geometric

limit of hyperbolic knot complements

Thm: Let $M \cong \mathbb{H}^3 / \Gamma$ hyperbolic, admitting
 $i: M \hookrightarrow S^3$ embedding & $\{M_i\}$ π_1 -injective
exhaustion. Then exists sequence $C_i \subseteq S^3$
of Cantor sets such that:

i) $N_i = S^3 \setminus C_i$ is hyperbolizable

ii) $N_i \xrightarrow{\text{geom}} M$

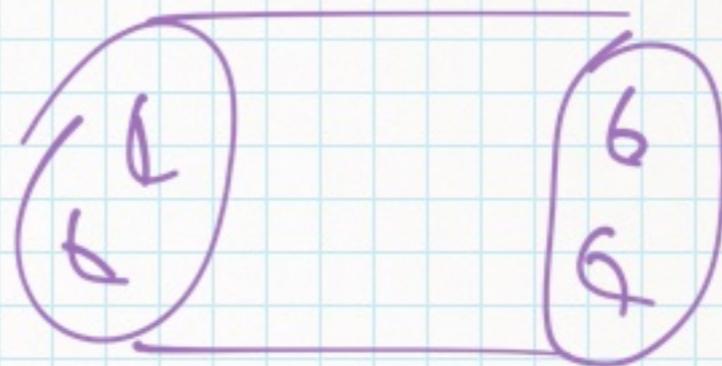
$R > 0, \epsilon > 0$

$f_i: B_R(p) \subseteq M \hookrightarrow N_i$

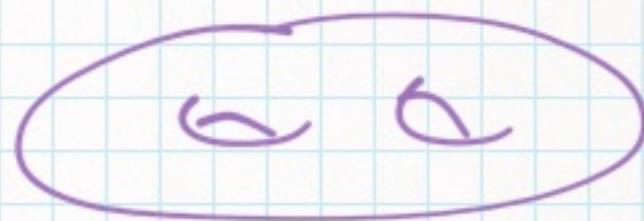
is $(1+\epsilon)$ -bilipschitz

Examples:

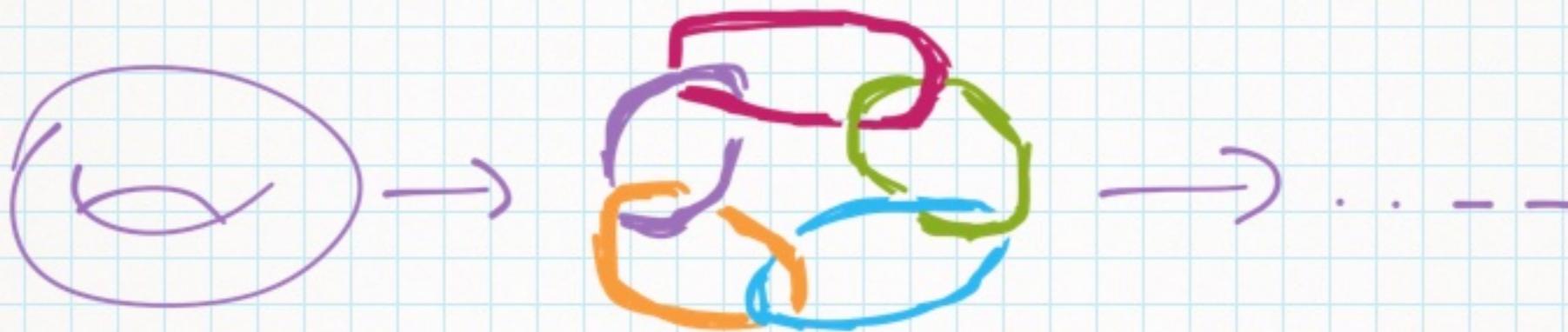
•) $M = \Sigma \times \mathbb{R}$



•) M handlebody

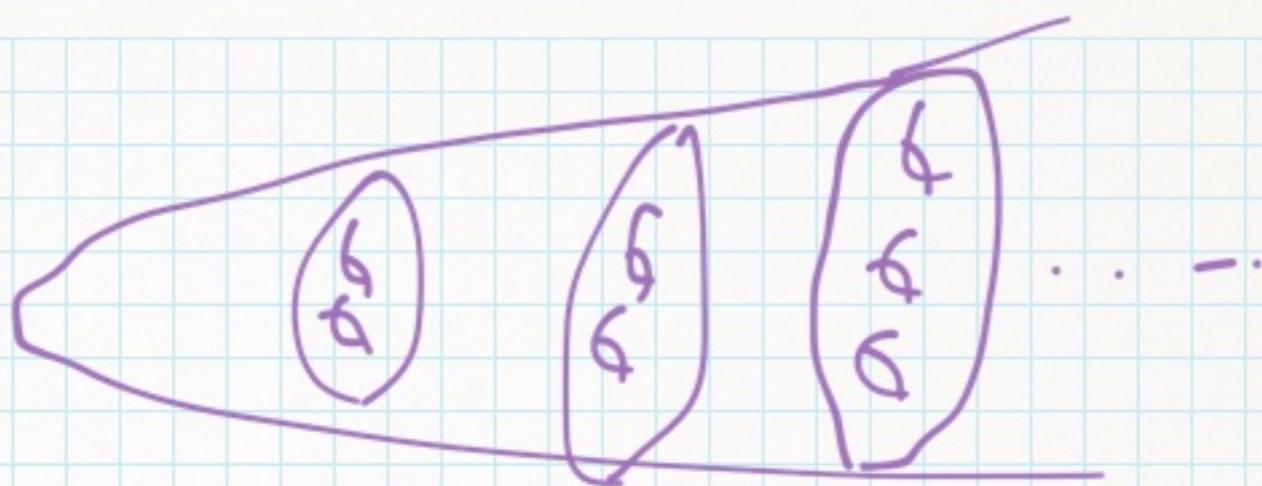


•) Antoine's necklace

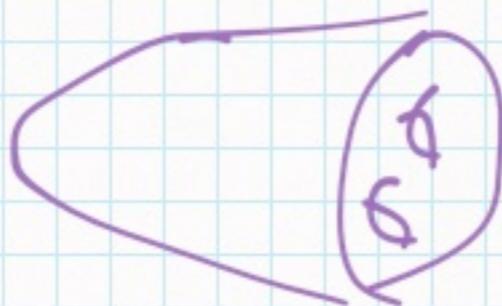


Sketch

$M =$

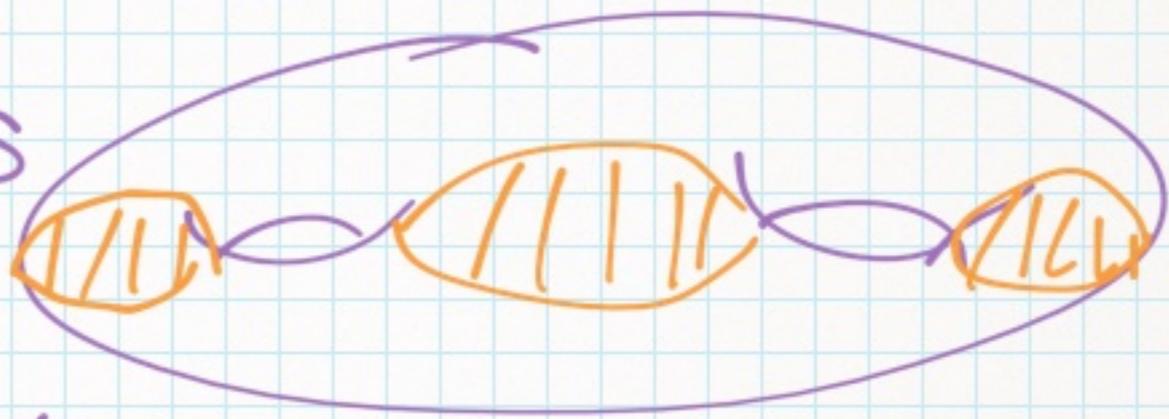


1) Reduce
 $M = \text{int}(\bar{M})$
 \bar{M} compact



2) Reduce to $S^3 \setminus M$: Union of handlebodies

3) Attach handlebodies
to compressing
disks in $S^3 \setminus M$



Step 1: $M = \text{int}(\bar{M})$, \bar{M} compact

M has a Π_1 -injective exhaustion

$\{M_i\}$

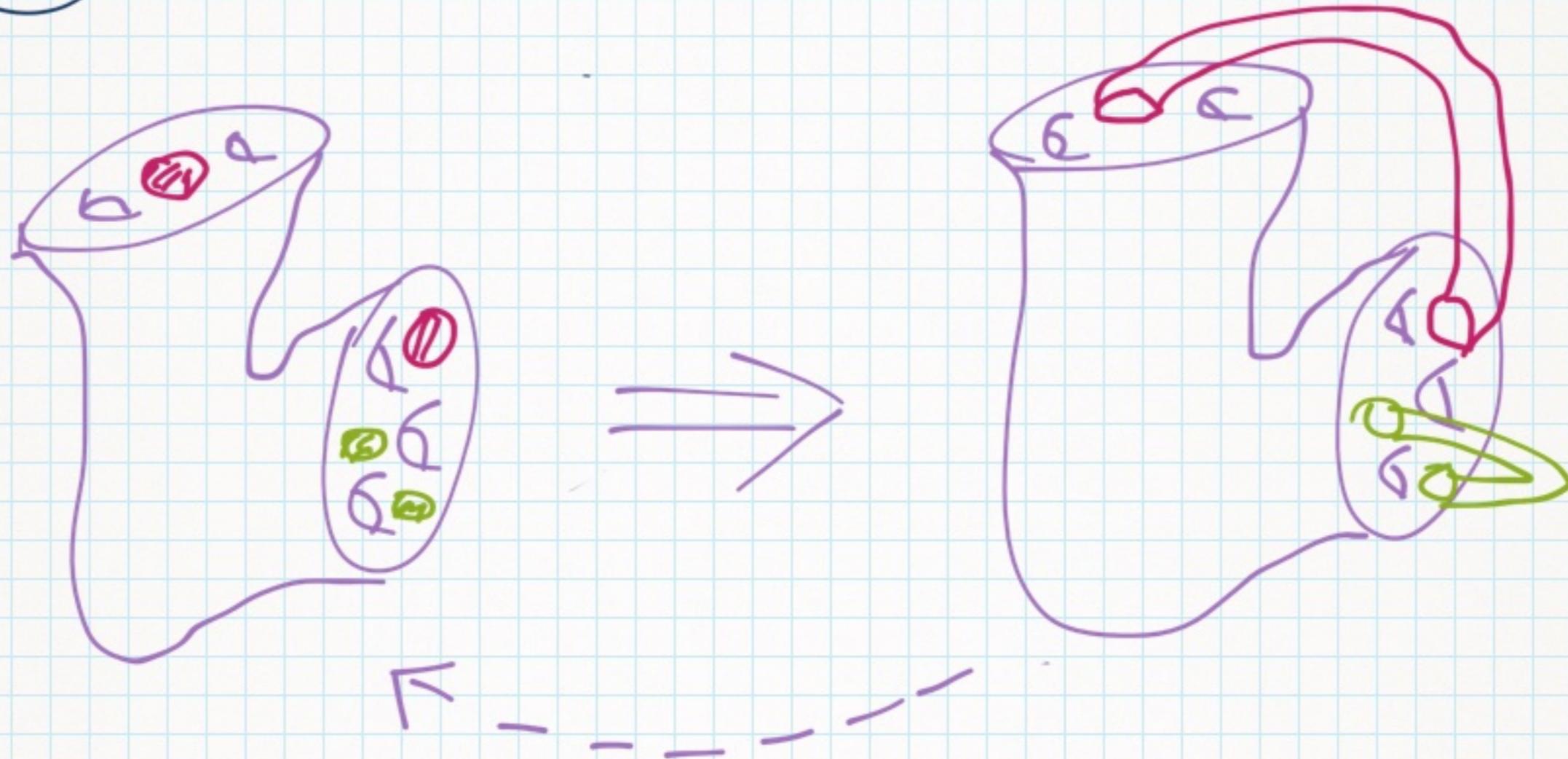
$\Rightarrow M_i$ is hyperbolic with

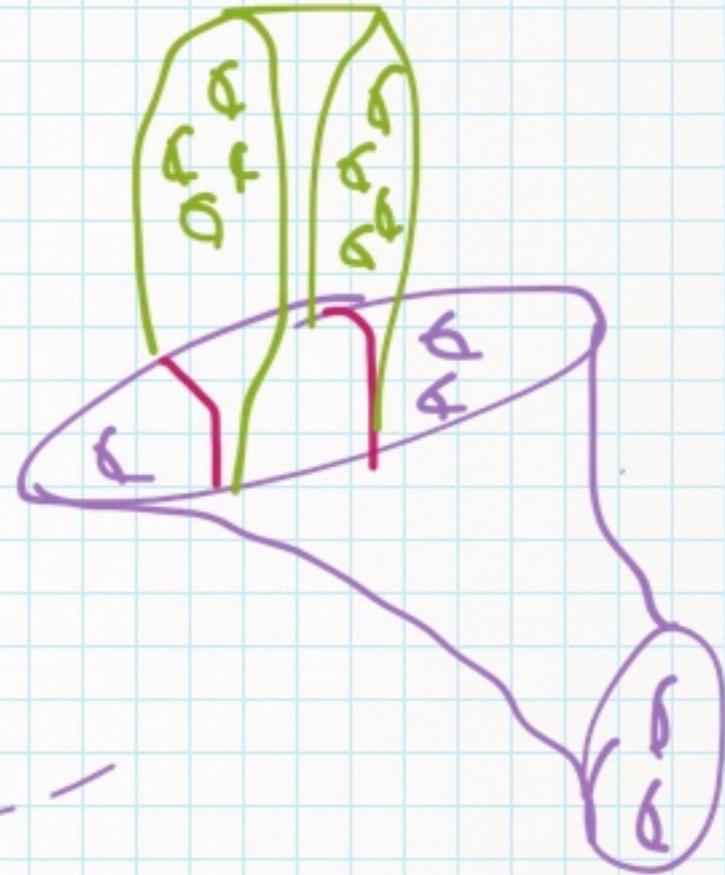
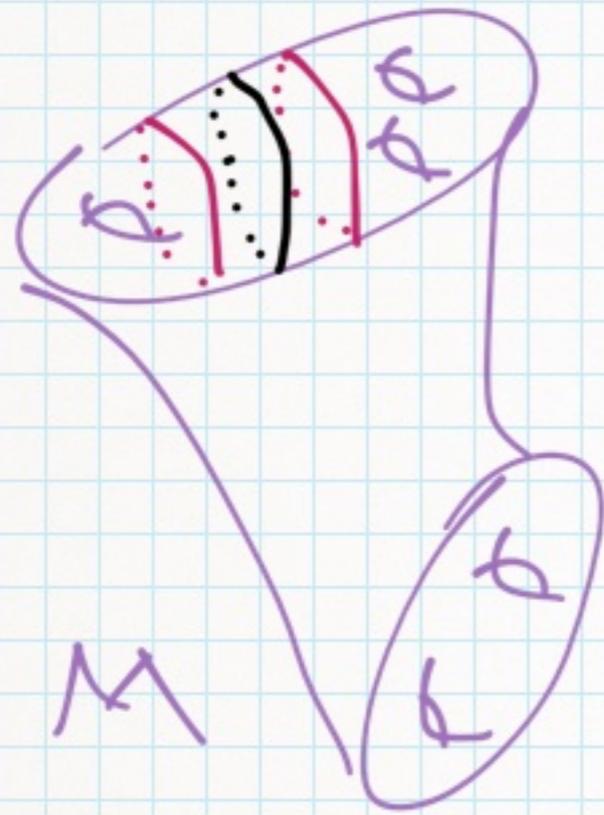
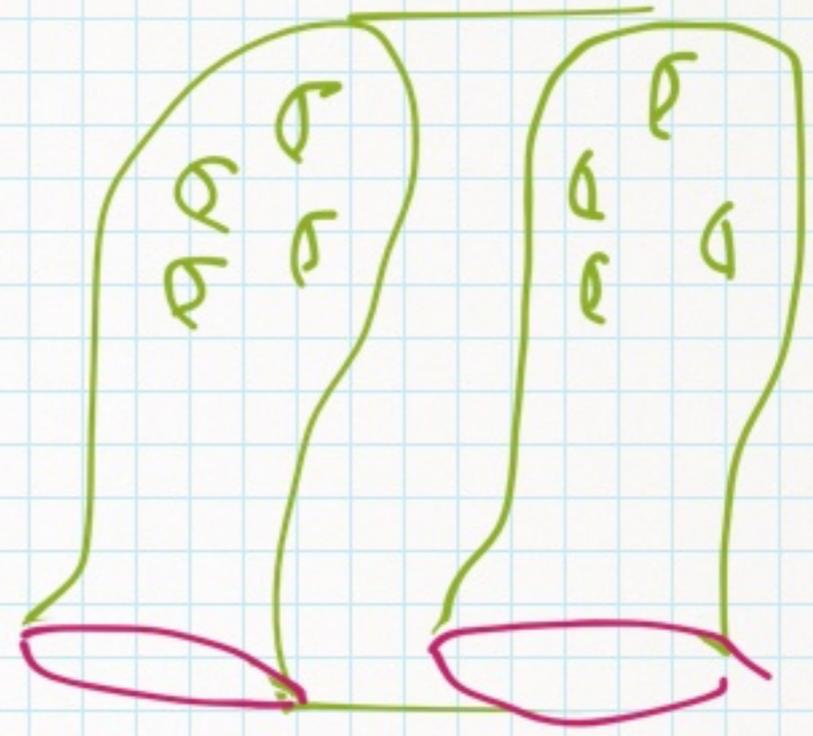
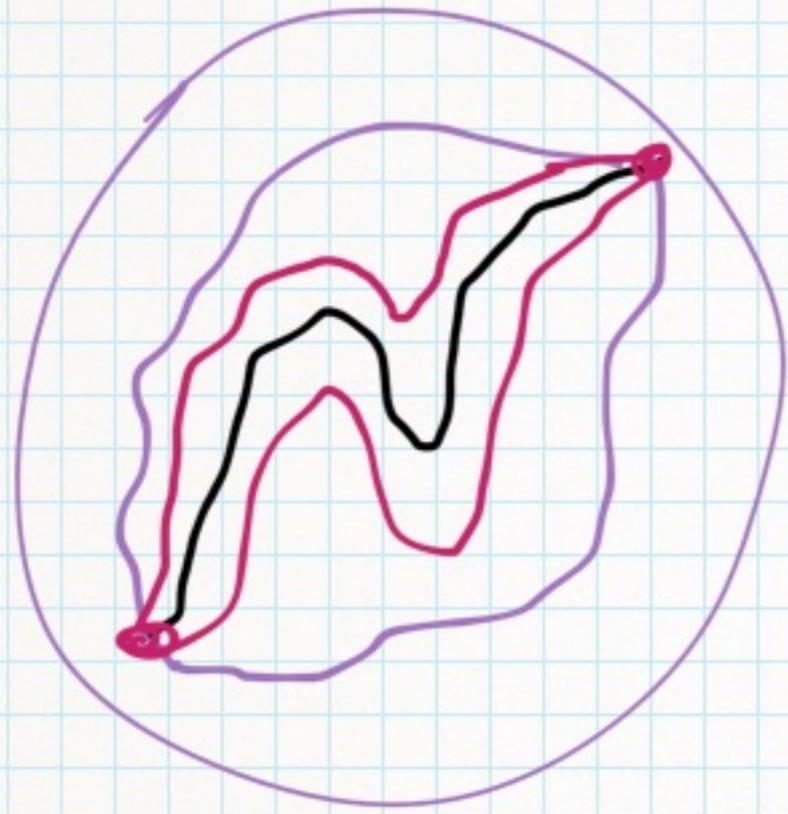
$$M_i \xrightarrow{\text{geom}} M.$$

By density, we can assume M_i

is convex co-compact

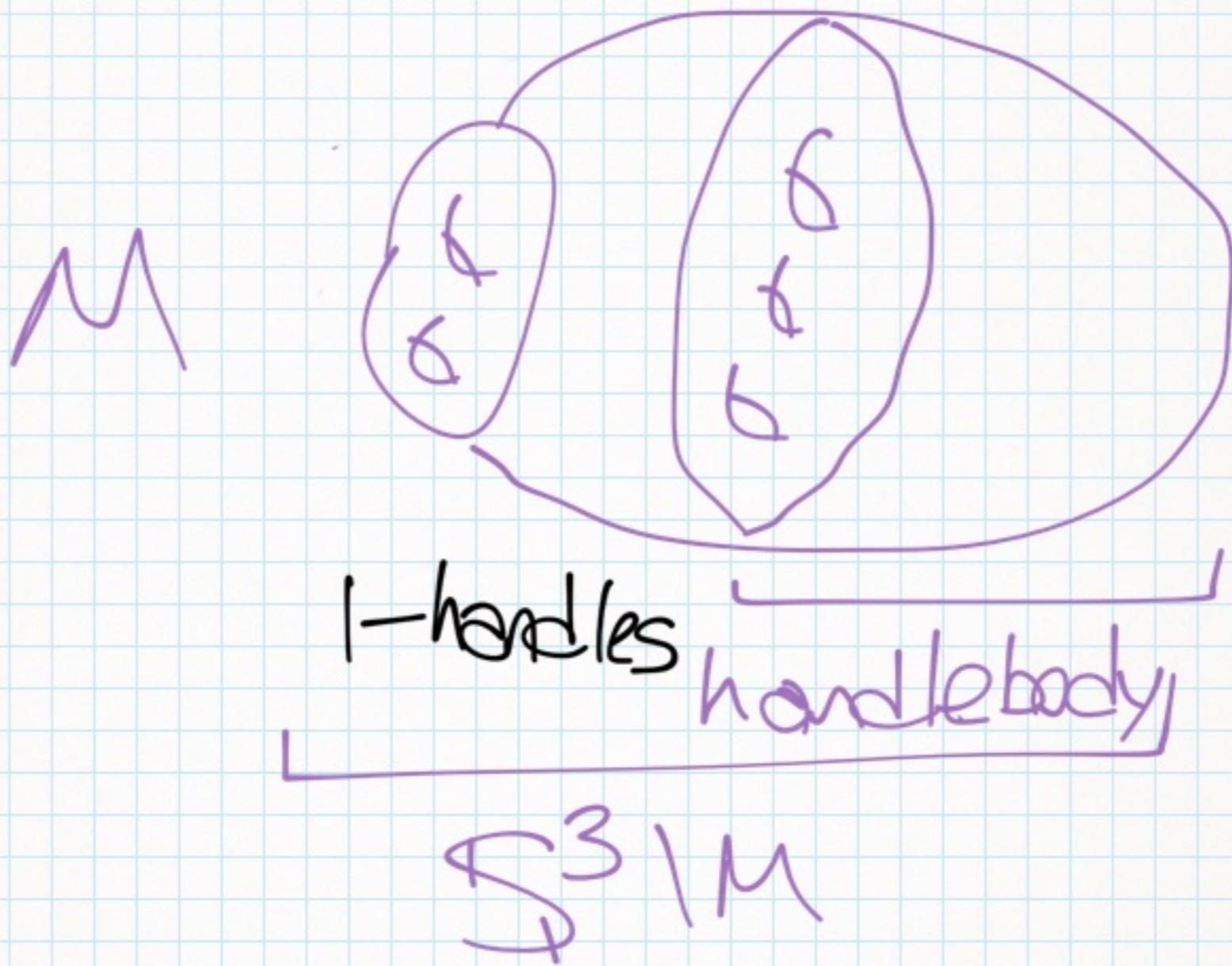
★ Combination Results





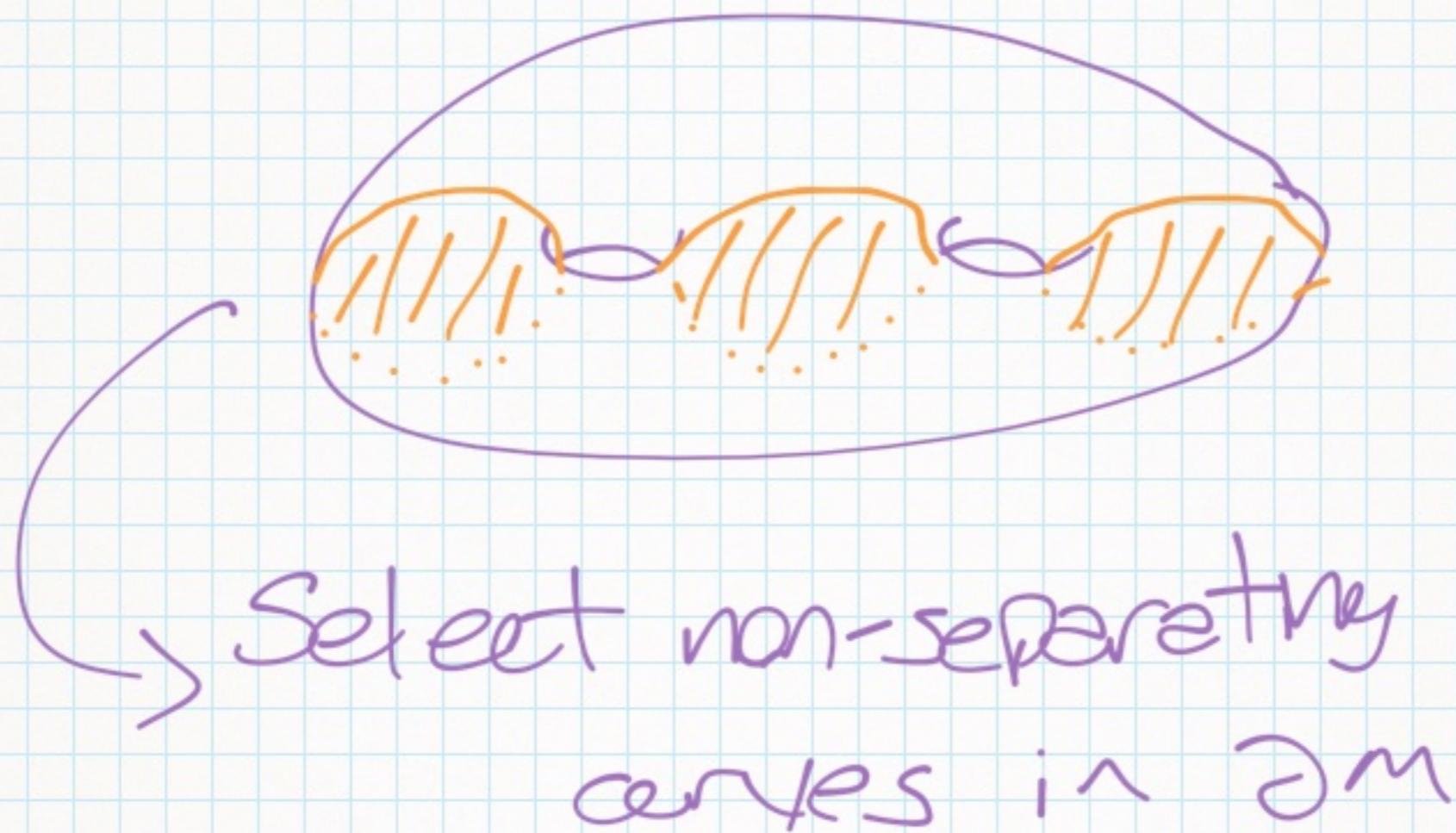
Step 2 : $S^3 \setminus M = \text{Union of handlebodies}$

Given $i: M \hookrightarrow S^3$, attach 1-handles
to obtain $M' \subseteq S^3$ so that $S^3 \setminus M'$
is a union of handlebodies

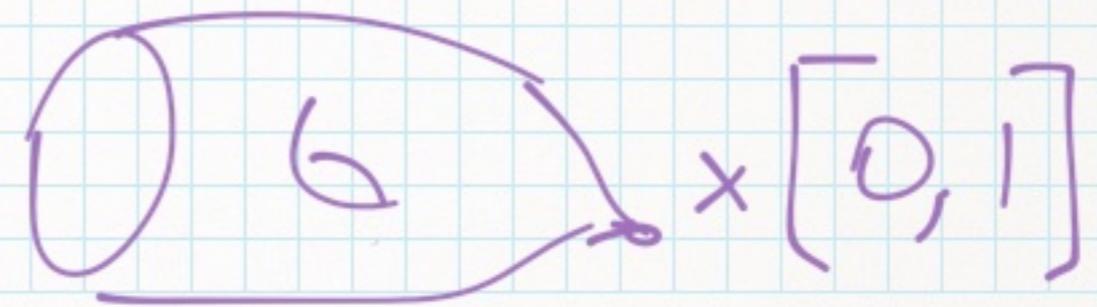
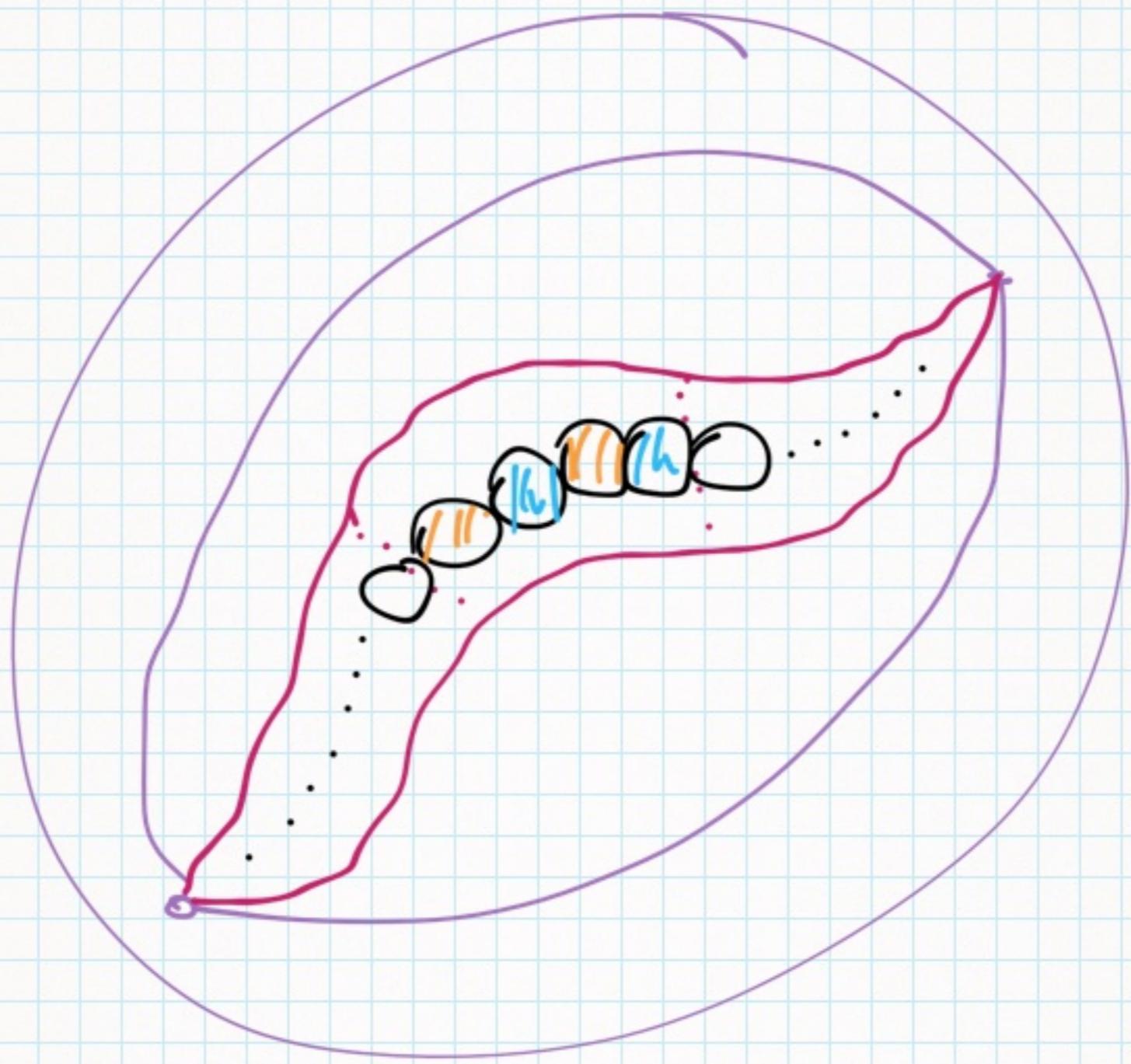


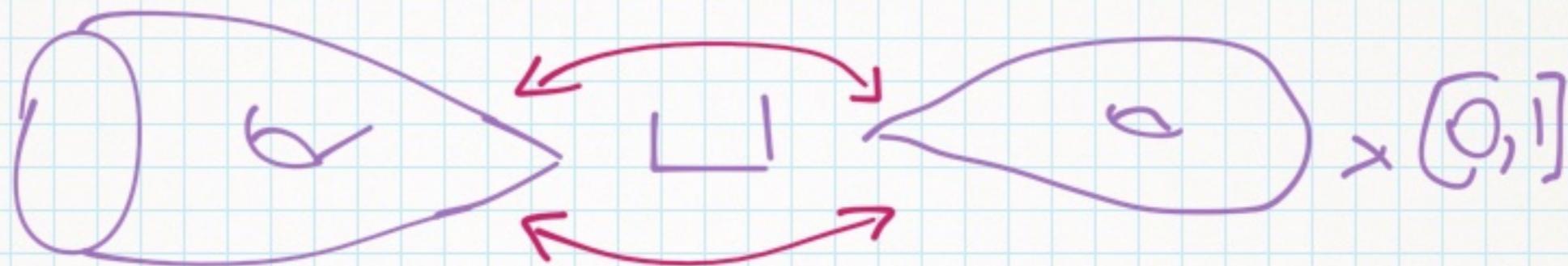
Step 3 Center set construction.

3.1) Select curves in ∂M that are π_1 -injective in M and compressible in S^3 .

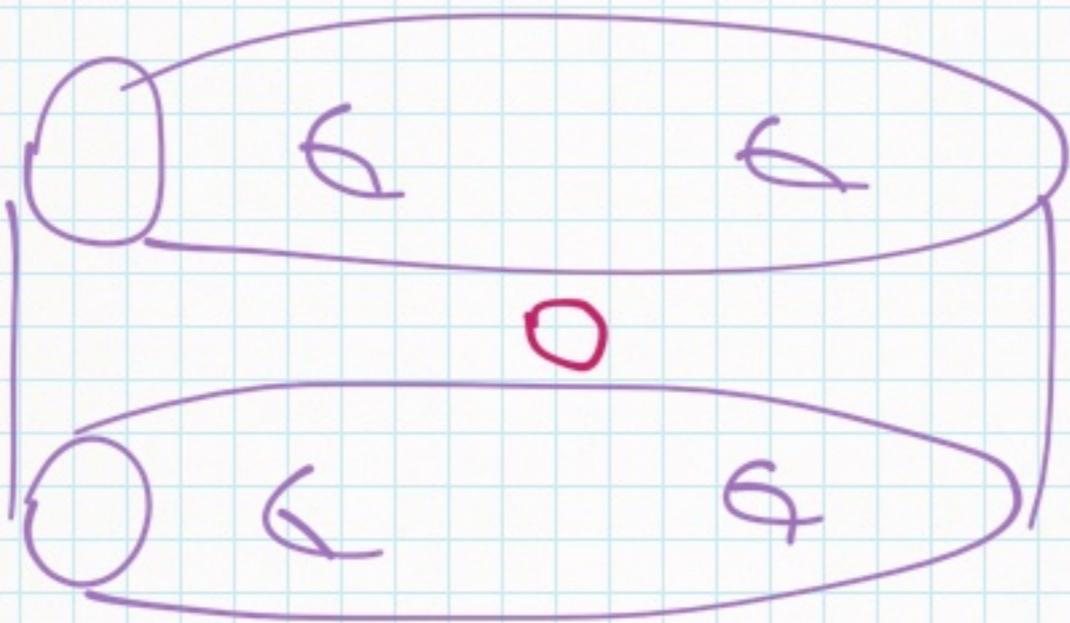


3.2) Handlebody models (to attach)

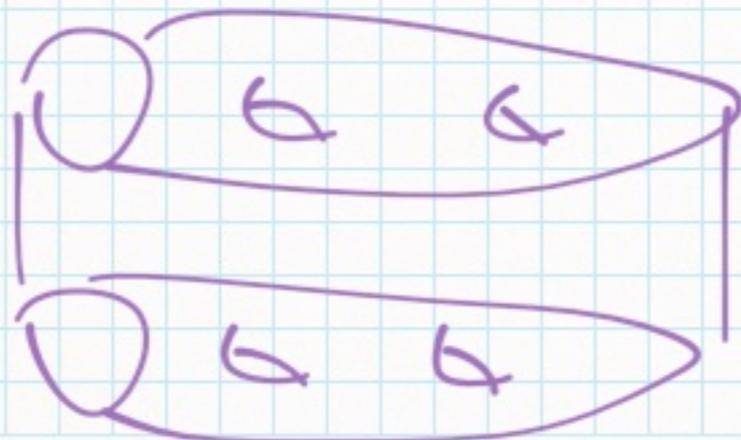




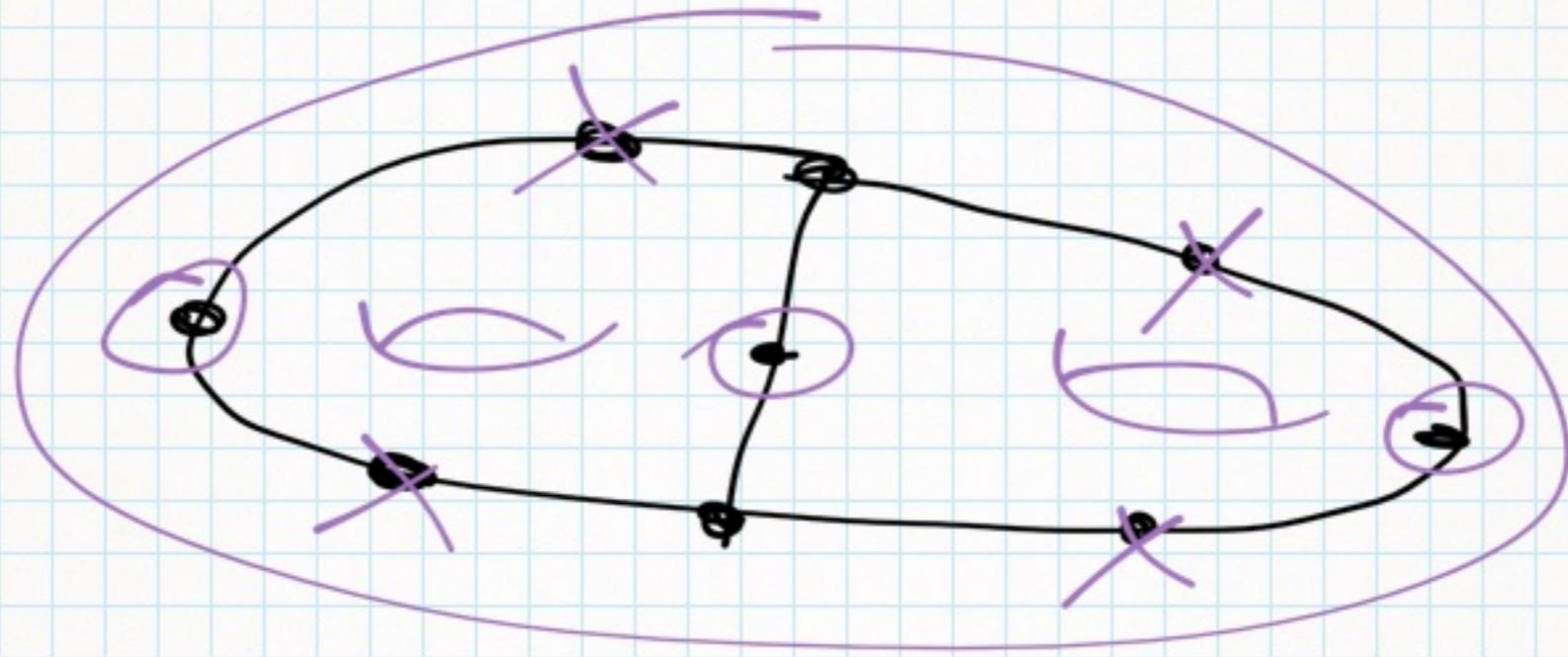
↓



↓ Dehn filling



3.3) See that diameter of the complement goes to 0.



Subdivide nerve to get

$$\text{Diameter}(M_{n+1}) \leq \frac{1}{2} \text{Diameter}(M_n)$$

(component-wise)

Conclusions:

- Cantor set complement are dense among $i: M \hookrightarrow \mathbb{S}^3$
 - Construct Cantor sets with
 -) Arbitrarily large geodesic balls
 -) Compact regions arbitrarily close to compact regions
- Of Schottky or Quasifuchsian

Thanks!!!