MATH 720, PROBLEM SET 5, DUE DEC 16

1. Problem 1

Let V be a symplectic vector space. Consider the Weyl algebra $W(V)$. Note that $\{\pm 1\}$ acts on $W(V)$ by automorphisms, the action comes from the action by changing the sign on V.

1) Show that the algebra of invariants $W(V)^{\{\pm 1\}}$ is a filtered quantization of $\mathbb{C}[\mathbb{O}]$, where $\mathbb{O}$ is the minimal nilpotent orbit in $\mathfrak{g} = \mathfrak{sp}(V)$.

2) Show that the natural action of $\mathrm{Sp}(V)$ on $W(V)$ is Hamiltonian, and moreover, the image of the quantum comoment map lies in $W(V)^{\{\pm 1\}}$.

3) Show that the resulting homomorphism $\Phi : U(\mathfrak{g}) \to W(V)^{\{\pm 1\}}$ is surjective.

The next two problems use the notation from Lecture 21.

2. Problem 2

Now let $\mathbb{O}$ be any orbit in $\mathfrak{g} = \mathfrak{sl}_n$ so that $T^*(G/P) \to \overline{\mathbb{O}}$ is a resolution of singularities for a suitable resolution P. Show that for any quantization parameter λ, the quantum comoment map $U(\mathfrak{g}) \to \Gamma(\mathcal{D}_\lambda)$ is surjective. *Hint: $\overline{\mathbb{O}}$ is normal.*

3. Problem 3

This is the last remark in Section 1.2 of Lecture 21. Show that the algebra $\Gamma(\mathcal{D}_{\mathfrak{z}})$ is free over $\mathbb{C}[\mathfrak{z}]$. *Hint: show that the Rees algebra $R_\hbar(\Gamma(\mathcal{D}_{\mathfrak{z}}))$ is free over $\mathbb{C}[\mathfrak{z}, \hbar]$ by relating it to the global sections of a suitable sheaf on G/P and using that $\mathbb{C}[Y_{\mathfrak{z}}]$ is free over $\mathbb{C}[\mathfrak{z}]$.*