

D-MODULES, HOMEWORK 2

Problem 1. Let X be a smooth affine curve. Prove that every irreducible $D(X)$ -module is holonomic.

Problem 2. Let $X = \mathbb{A}^n$ and let L be a lagrangian subspace in $\text{Span}_{\mathbb{C}}(x^1, \dots, x^n, \partial^1, \dots, \partial^n)$. Compute the singular support and the characteristic cycle of $D(\mathbb{A}^n)/D(\mathbb{A}^n)L$.

Problem 3. Let $a \in \mathbb{C} \setminus \mathbb{Z}$. Consider the $D(\mathbb{A}^1)$ -module $\mathbb{C}[x^{\pm 1}]x^a$ that is identified with $\mathbb{C}[x^{\pm 1}]$ as a $\mathbb{C}[x]$ -module and ∂ acts on $x^n x^a$ via $\partial(x^n x^a) = (n + a)x^{n-1}x^a$ (here x^a can be treated as a formal symbol or as an actual multi-valued function). Prove that the $D(\mathbb{A}^1)$ -module $\mathbb{C}[x^{\pm 1}]$ is irreducible and compute its singular support and characteristic cycle.