

GRADUATE SEMINAR ON D-MODULES

1. INTRODUCTION

D-modules, i.e., modules over (sheaves of) algebras of differential operators on smooth algebraic varieties play a very important role in Algebraic geometry and Representation theory. The goal of this seminar is to learn basics about D-modules.

2. FORMAT

Students prepare lecture with some advice from the faculty.

3. PRELIMINARY LIST OF TOPICS

Below is an overly optimistic preliminary plan.

- 1) Basics: differential operators on smooth varieties and modules over them.
- 2) Support, Kashiwara's lemma. Singular support. Bernstein inequality. Holonomic D-modules.
- 3) The six functors. Preservation of holonomicity.
- 4) Classification of simple holonomic modules. Regular holonomic modules and the Riemann-Hilbert correspondence.
- 5*) Hodge D-modules. Applications to algebraic geometric geometry, e.g., Kollar's decomposition theorem and the Kodaira vanishing theorem.
- 6*) Connections to Representation theory of semisimple Lie algebras: the Beilinson-Bernstein localization theorem and Kazhdan-Lusztig conjectures.

4. PREREQUISITES

Some algebraic geometry, in particular, the participants should be comfortable with (smooth) algebraic varieties, tangent spaces/ bundles, etc. Some knowledge of derived categories should be useful for various topics, such as 3). 5) will require a more substantial algebro-geometric background, and for 6) one will need more background from Representation theory.

5. REFERENCES

These include:

- 1) R. Hotta, K. Takeuchi, T. Tanisaki, *D-modules, perverse sheaves, and representation theory*. Progress in Mathematics, 236. Springer.
- 2) Notes by V. Ginzburg or by J. Bernstein.
- 3) M.-H. Saito, *Applications of Hodge modules*, available here.
- 4) S.C. Coutinho, *A primer of algebraic D-modules*, Cambridge University Press, 1995.
- 5) and more.