# INVARIANT HOMEWORK 1, DUE FEB 5

For simplicity, the base field below is $\mathbb{C}$. By $G$ we denote a reductive algebraic group.

## Problem 1

Let $V$ be a faithful finite dimensional rational representation of $G$ ("faithful" means that the homomorphism $G \to \mathrm{GL}(V)$ is injective). Suppose that every $G$-orbit in $V$ is closed. Show that $G$ is finite. Hint: look at fibers of the quotient morphism $\pi : V \to V//G$.

## Problem 2

Let $X$ be an affine variety with a $G$-action, $Y$ a variety, and $\psi : X \to Y$ be a $G$-invariant morphism. Show that $\psi$ uniquely factors through $X//G$ (in Lecture 3 we have proved this in the case when $Y$ is affine).