INVARIANT HOMEWORK 2, DUE FEB 21

For simplicity, the base field below is $\mathbb{C}$.

Problem 1

Let d, n be positive integers $d > 1$, $G = \mathrm{GL}_{nd}$ and γ be the diagonal matrix

$$\mathrm{diag}(1, \ldots, 1, \epsilon, \ldots, \epsilon, \epsilon^2, \ldots, \epsilon^2, \ldots, \epsilon^{d-1}, \ldots, \epsilon^{d-1}),$$

where ϵ is a primitive d-th root of 1 and each ϵ^i occurs n times. Let $\theta = \mathrm{Ad}(\gamma)$. For the corresponding pair $(G_0, \mathfrak{g}_1)$ identify the Cartan subspace of $\mathfrak{g}_1$ with $\mathbb{C}^n$ and the Weyl group with $S_n \ltimes \mu_d^n$, where μ_d is the group of d-th roots of 1.[1]

Problem 2

For each $n > 1$ construct an example of the following: a vector space V acted on by $\mathbb{C}^\times$ and a $\mathbb{C}^\times$-stable divisor $D \subset V$ such that $\dim(V/\!/\mathbb{C}^\times) = n$ and the image of D in $V/\!/\mathbb{C}^\times$ is a single point.

[1]In particular, for $d > 2$ we get a genuine complex reflection group