INVARIANT HOMEWORK 4, DUE APR 17

Problem 1

Let G be a reductive group acting on an irreducible affine variety X. To a reductive subgroup $H \subset G$ we assign the locus $(X//G)_{(H)}$ consisting of all points such that the corresponding closed orbit is isomorphic to G/H. Prove that there is a unique up to conjugacy subgroup $H \subset G$ such that $(X//G)_{(H)}$ is nonempty and Zariski open in $X//G$. *Hint: the Luna slice theorem.*

Problem 2

Let V be a finite dimensional rational representation of G and H is the reductive subgroup of G determined for the action of G on V as in the previous problem. The goal of this problem is to establish the following generalization of the Chevalley restriction theorem: $V^H//N_G(H) \xrightarrow{\sim} V//G$.

a, 1pt) Construct a natural morphism $V^H//N_G(H) \to V//G$.

b, 3pts) Show that this morphism is birational.

c, 3pts) Show that the preimage of $\pi_G(0)$ coincides with $\pi_{N_G(H)}(0)$.

d, 2pts) Use c) to show that the morphism is finite.

e, 1pt) Prove that this morphism is an isomorphism.