Classical stuff (warning: requires more prerequisites than the class).
Below char $\mathbb{F}=p>0$, $G=SL_n(\mathbb{F})$

- Kempf vanishing theorem (the higher derived functors $R^i\mathrm{Ind}_B^G$ vanish on $\mathbb{F}_{w_0\lambda}$ $\Leftrightarrow$ cohomology vanishing result): [J], Sec 4 in Ch. II.
- Weyl character formula for $\mathrm{ch}\, M(\lambda)$: [J], Sec 5 in Ch. II.
- Steinberg tensor product theorem: [J], Sec 3 in Ch. II.

Character formulas for simples: since we know $\mathrm{ch}\, M(\lambda)$, we reduce the computation to expressing $\mathrm{ch}\, L(\mu)$ via $\mathrm{ch}\, M(\lambda)$'s as in Section 2 of Lec 16.

Guess: for $\mu\in\Lambda_+^1$ these coefficients are values at -1 of "parabolic affine KL polynomials" as long as p is "sufficiently large."

Results: • True for $p\gg n$ w/o explicit bound

Andersen-Jantzen-Soergel: "Representations of quantum groups at pth root of unity & of semisimple groups in characteristic p: independence of p." Astérisque N 220 (1994), 321 pp.

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Kazhdan-Lusztig "Tensor structures arising from affine Lie algebras." Parts I-IV in J. Amer. Math. Soc. 1993 & 1994: >200 pp

– relating representations of affine Lie algebras to those of quantum groups

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Kashiwara-Tanisaki "Kazhdan-Lusztig conjecture for affine Lie algebras with negative level", 2 papers in Duke 1995 & 1996.

In 2000's more direct approaches were found (e.g. Bezrukavnikov & collaborators). They will be mentioned later.

Absolutely enormous explicit bound on p for the conjectured character formula to hold (p should be of order n^{n^2})
Fiebig "Sheaves on affine Schubert varieties, modular representations, and Lusztig's conjecture", J. Amer. Math. Soc. 2011.

Williamson "Schubert calculus & torsion explosion" J. Amer. Math. Soc. 2017: for the conjectured character formula to hold p must grow at least exponentially in n.