REPRESENTATION THEORY, HINTS TO PROBLEM SET 2

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Problem 1. . Hint for 3). Reduce the problem to checking a similar statement for $S(\mathfrak{g})$. Look at the component of degree $(?, 1)$.

Hint for 4): there's an associative algebra homomorphism $U(\mathfrak{g}) \to \mathfrak{g}$.

Problem 2. It may be useful to consider vectors annihilated by e ad the action of the Casimir operator.

Problem 3. Look at the action of $\begin{pmatrix} 1 & z^2 x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} 1 & x \\ 0 & x \end{pmatrix} \begin{pmatrix} z^{-1} & 0 \\ 0 & z \end{pmatrix}$.