PRE-SEMINAR PROGRAM, 2019

1. Finite type

1.1. **Quick reminder on reductive groups and Lie algebras,** [H1, OV, Sp] **or** [J, Section II.1]**.**

1.1.1. *Definitions and examples.*

1.1.2. *Borel subgroups, maximal tori, Weyl group.*

1.1.3. *Root systems and root data.*

1.1.4. *Flag variety, Bruhat decomposition.*

1.1.5. *Finite dimensional representation theory over* $\mathbb{C}$*.*

1.2. **BGG category $\mathcal{O}$.**

1.2.1. *Definition, Verma modules, simple modules,* [G, Section 1].

1.2.2. *Harish-Chandra isomorphism and block decomposition. Finite length. Structure of* K_0*,* [G, Sections 2,3].

1.2.3. *Duality, dual Verma modules,* [G, Section 3].

1.2.4. *Projective functors and projective objects, BGG reciprocity, order,* [G, Section 4],[R].

1.2.5. *Highest weight structure and tilting objects,* [K],[H2, Chapter 11].

1.2.6. *Parabolic categories $\mathcal{O}$: parabolic Verma modules, structure of* K_0*,* [H2, Chapter 9].

1.3. **Kazhdan-Lusztig theory.**

1.3.1. *Hecke algebras of Coxeter groups,* [T], [So, Section 2].

1.3.2. *Bar involution and Kazhdan-Lusztig basis,* [So, Section 2].

1.3.3. *Variations: spherical and anti-spherical modules,* [So, Section 3].

1.3.4. *Application: multiplicities in category* $\mathcal{O}$*,* [H2, Chapter 8].

1.4. **Perverse sheaves and applications.**

1.4.1. *Sheaves of vector spaces, local systems,* [PS, Stefan's talk].

1.4.2. *Pull-back, push-forward, internal Hom, and shriek-pullback functors* [PS, Stefan's and Roger's talks].

1.4.3. *Constructible sheaves and constructible derived category,* [PS, Balasz's and Hyungseop's talks].

1.4.4. *Dualizing sheaf, Verdier duality,* [PS, Hyungseop's talk].

1.4.5. *Perverse sheaves,* [PS, Kathlyn's and Yehao's talks].

1.4.6. *Case of flag varieties,* [PS, Kostya's talk].

2. Affine type

2.1. **Affine Weyl groups,** [So, Section 4]**.**

2.1.1. *Affine Dynkin diagrams.*

2.1.2. *Lattice presentation, length function.*

2.1.3. *Alcoves, generic order.*

2.1.4. *Extended affine Weyl group.*

2.2. **Modular representations of reductive groups.**

2.2.1. *Dual Weyl modules, classification of simples,* [J, Section II.2].

2.2.2. *Frobenius kernels and their irreducible representations. Steinberg tensor product decomposition,* [J, Sections II.3, II.9].

2.2.3. *Kempf vanishing and Weyl character formula,* [J, Sections II.4, II.5].

2.2.4. *p-Weyl group action, linkage,* [J, Section II.6].

2.2.5. *Projective functors,* [J, Section II.7].

2.2.6. *Highest weight structure and tilting modules.*

2.3. **Affine representation theory,** [Z].

2.3.1. *Loop groups and their parahoric subgroups.*

2.3.2. *Affine grassmanians and flag varieties.*

2.3.3. *Affine categories $\mathcal{O}$.*

2.3.4. *Geometric Satake equivalence.*

References

[G] D. Gaitsgory, *Geometric representation theory.* http://www.math.harvard.edu/ gaitsgde/267y/catO.pdf
[H1] J. Humphreys, *Linear algebraic groups*, GTM 21.
[H2] J. Humphreys, *Representations of Semisimple Lie Algebras in the BGG Category O.* GTM 94.
[J] J.C. Jantzen, *Representations of algebraic groups.* Pure and Applied Mathematics, 131.
[K] D. Kalinov, *Category $\mathcal{O}$ and its basic properties*, Seminar notes available at https://web.northeastern.edu/iloseu/Kalinov_OSBim.pdf
[OV] A. Onishchik, E. Vinberg, *Lie groups and algebraic groups*, Springer series in Soviet Mathematics.
[PS] Perverse seminar by various speakers, http://www.math.toronto.edu/ jkamnitz/seminar/perverse/perverse.html.
[So] W. Soergel, *Kazhdan-Lusztig polynomials and a Combinatoric for tilting modules.* Repres. Theory, 1 (1997), 83-114.
[R] C. Ryba, *Tensoring with finite dimensional in category $\mathcal{O}$.* Seminar notes available at https://web.northeastern.edu/iloseu/Ryba_OSBim.pdf
[Sp] T.A. Springer, *Linear algebraic groups.* Progr. Math. 9.
[T] B. Tsvelikhovsky, *Soergel bimodules, Hecke algebras and Kazhdan-Lusztig basis.* Seminar notes available at https://web.northeastern.edu/iloseu/Boris_OSBim.pdf
[Z] X. Zhu, *An introduction to affine Grassmannians and the geometric Satake equivalence.* arXiv:1603.05593.