HW1, extra-credit problem. Let $f_1, \ldots, f_k$ be a finite collection of $G$-invariant elements generating $\mathbb{C}[V][V]^G$. The common set of zeros of $f_1, \ldots, f_k$ is $\{0\}$ if $G$ the $G$-invariant elements separate the orbits. So the preimage of $0 \in \mathbb{C}^k$ under $V \rightarrow \mathbb{C}^k$ is $0$. It follows that this morphism is finite. Therefore all fibers are finite. The morphism is also $G$-invariant.

It follows that all $G$-orbits are finite. But $G = GL(V)$, hence $G^0 = \{1\}$ and $G$ is finite.