Problem 1.1 (Krull-Schmidt theorem): Let A be an associative algebra, V be its finite dimensional module. Let $V = V_1 \oplus V_2 \oplus \ldots \oplus V_k = V_1' \oplus V_2' \oplus \ldots \oplus V_\ell'$ be two decompositions into indecomposable modules. Then $k = \ell$ and, after rearranging summands, we get $V_i \cong V_i'$.

Problem 1.2: Show that the real roots of a quiver Q are precisely elements of the form $w\alpha_i$, where w is in the Weyl group $W(Q)$. Show that $v \in \mathbb{Z}_{\geqslant 0}^{Q_0}$ is an imaginary root if and only if $W(Q)v \subset \mathbb{Z}_{\geqslant 0}^{Q_0}$.

Problem 1.3: Verify the three theorems on the indecomposable representations (=Kac's theorem) in the following cases:
  a) Type A Dynkin quiver with orientation from left to right.
  b) Type D_4 Dynkin quiver with all arrows towards the central vertex.
  c) Cyclic quiver with n vertices and cyclic orientation.
  d*) Cyclic quiver with two vertices $1, 2$ and both arrows going from 1 to 2.

Problem 1.4: Provide an example (of an algebraic group G acting on a variety X) when the number of parameters needed to describe orbits of maximal dimension is not the same as the maximal number of parameters.

Problem 1.5: Suppose a quiver Q contains no oriented cycles. Then the dimension of an irreducible representation is a simple root and the number of isomorphism classes of irreducible representations coincides with the number of vertices.

Problem 2.1: Show that the dimension of an indecomposable representation is a root even if that dimension is not primitive.

Problem 3.1 (the main lemma for nilpotent matrices). Let λ be a Young diagram and let λ' denote the diagram obtained from λ by removing the first column. Set $n := |\lambda|, n' := |\lambda'|$. Let V, V' be vector spaces of dimensions n, n' respectively. Let $\mathcal{O}_\lambda$ be the conjugacy class of nilpotent matrices of size n with Jordan form corresponding to λ (the parts of λ are the sizes of blocks) and let $\mathcal{O}_{\lambda'}$ have a similar meaning.
  Consider the space $X := \text{Hom}(V, V') \oplus \text{Hom}(V', V)$ and morphisms $\mu_1 : X \to \text{End}(V), \mu_2 : X \to \text{End}(V')$ given by $\mu_1(A, B) = BA, \mu_2(A, B) = AB$ (where $A \in \text{Hom}(V, V'), B \in \text{Hom}(V', V)$).
  a) Prove that $\mu_1(\mu_2^{-1}(\overline{\mathcal{O}_{\lambda'}})) = \overline{\mathcal{O}_\lambda}$.
  b) Show that the following two conditions are equivalent:
   (i) $BA \in \mathcal{O}_\lambda$.
   (ii) $AB \in \mathcal{O}_{\lambda'}, B$ is injective, A is surjective.

Problem 4.1. Show that the vector $(\dim N_i)_{i=0}^r$ (where $N_0, \ldots, N_r$ are the irreducible representations of a Kleinian subgroup Γ) is the indecomposable imaginary root of the corresponding McKay quiver (no case-by-case, please!).