Introduction to Functional Analysis (325b/525b)

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Topics covered and time allocated

Number of lectures marked by (n). A * indicates a non-examinable topic. The following is a tentative schedule, based on Bollobás's book 'Linear Analysis'. I've left a couple of spare lectures for flexibility/additional topics.

Inequalities(1): AM-GM. Jensen's Theorem. Cauchy-Schwarz. Hölder. Minkowski.

Normed spaces and bounded linear operators(3): Topologies, norms, completeness, separability. Examples. Bounded linear operators and their properties. Examples. Completions of spaces. Schauder bases. Quotient spaces. The Riesz-Fischer Theorem.

Linear functionals(2): Definitions and first properties. The Hahn-Banach Theorem and applications. Convex sets in Banach spaces.

Finite dimensional spaces(2): All norms are equivalent. Characterization by compactness of unit ball. John's Theorem*.

The Baire Category Theorem(2): The Theorem. Definition of category. Principle of uniform boundedness. Banach-Steinhaus Theorem. Open mapping Theorem. Inverse mapping Theorem. Closed graph Theorem. Examples.

Continuous functions on compact spaces(3): Tietze-Urysohn extension Theorem. Compactness properties and equicontinuity. The Arzelà-Ascoli Theorem and applications. First look at Banach algebras. Stone-Weierstrass Theorem and applications.

The contraction mapping theorem (1). The theorem and examples.

Weak topologies and duality(2) Definitions. The Banach-Alaoglu Theorem. Weak-$*$ topology. The Krein-Millman Theorem*.

Euclidean spaces and Hilbert spaces(2). Geometry of Euclidean spaces. Projections. Riesz representation Theorem.

Orthonormal systems(1): Definitions and examples. The Fourier transform. Bessel's inequality. Parseval's identities.

Adjoint operators(1) : Definitions. C^*-algebras. The Gelfand-Naĭmark Theorem*. Hermitian parts of operators, projections, unitary operators. Invertibility of a bounded linear operator on a Banach space.

The algebra of bounded linear operators(2): Spectrum, resolvent set, eigenvalues. Polynomial mapping of spectrum. Spectral radius formula. Gelfand-Mazur Theorem.

Compact operators on Banach spaces(2): Definition of compact operator. Hilbert-Schmidt and Trace class operators. The Spectral Theorem for Compact operators.

Prerequisites.

You need to have taken a class in Real Analysis and know some point set topology. Measure theory is needed only at few points where the results can be used as a 'black box'. I'll explain what I need at that time.

Times and locations

We meet Monday and Wednesday from 1.00pm to 2.15pm in LOM 206. I will have office hours from 2.30-4.30 on Mondays in my office 418 Dunham Labs, or by appointment.

How the class is graded

One midterm, worth 30%. Final worth 40%. There will be 6 homeworks worth 5% each.

Textbooks

We will be following very closely the book 'Linear Analysis (an introductory course)' by Béla Bollobás. It should be completely sufficient and I really like the presentation of this book.

There are many other books on Functional Analysis, in case you want another treatment. One very well known one is Rudin's 'Functional Analysis'.