AMTH/MATH 222 Final Exam Review

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Questions involving a calculation (show your working).

1. Find the general solution to the system of linear equations

$$\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 3 & 9 & 3 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}.$$

2. If it exists, find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

3. Find the LU decomposition of the matrix in question 2.

4. Find a basis for the 2D plane in $\mathbb{R}^4$ given by the equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ x_2 + x_3 &= 0. \end{aligned}$$

5. Put the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{pmatrix}$$

into reduced row echelon form.

6. Find the QR decomposition of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 0 & -2 & 3 \end{pmatrix}.$$

7. Find the eigenvectors and eigenvalues of the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 3 \end{pmatrix}.$$

8. Find the determinant of

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ -1 & -4 & -6 & 0 \\ 3 & 6 & 6 & 1 \\ 1 & -4 & -14 & 6 \end{pmatrix}.$$

9. Diagonalize the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Why is it possible to diagonalize A?

10. Calculate

$$\begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix}^{1024}.$$

11. Give an example of a 3×3 matrix that is not diagonalizable. Why?

12. Solve the difference equation

$$u_k = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} u_{k-1}, \quad u_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

What is the long term ($k \to \infty$) behavior of u_k?

13. Determine whether

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

is positive definite.

14. Find the singular value decomposition of

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

15. Compute the discrete Fourier transform of the vector

$$v = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

Which is the largest Fourier coefficient c_i?

True or False questions

Answer true or false: you should give a short but convincing explanation for your answer.

a. Row operations don't change the determinant.

b. For $n \times n$ matrices A and B, $|A + B| = |A| + |B|$.

c. $|A^{-1}| = |A|^{-1}$ if A is nonsingular $n \times n$.

d. A permutation matrix has determinant ± 1.

e. If all the cofactors of a matrix are nonzero, the determinant is nonzero.

f. Every $n \times n$ matrix with n distinct eigenvalues has n independent eigenvectors.

g. Every symmetric matrix has n distinct eigenvalues.

h. Every symmetric matrix has n independent eigenvectors.

i. Every diagonalizable matrix ($A = S\Lambda S^{-1}$) can be diagonalized using an orthogonal matrix S.

j. A matrix with only real eigenvalues is symmetric.

k. The matrices A and A^T have the same eigenvalues.

l. The matrices A and A^T have the same eigenvectors.

m. Eigenvectors of a real symmetric matrix with different associated eigenvalues are perpendicular.

n. Every matrix with only one eigenvalue, equal to 1, is similar to the identity.

o. The sum of two positive definite matrices is positive definite.

p. Every matrix has a singular value decomposition.

q. The pivots of a square matrix are the same as its eigenvalues.

r. The nth power of a real symmetric matrix either goes to 0 or is unbounded as $k \to \infty$.

s. The eigenvalues of AB are the products of eigenvalues of A and eigenvalues of B.

t. A matrix with some eigenvalue 0 is singular.

u. The trace of a square matrix is the same as the sum of its eigenvalues.

v. The eigenvalues of an upper triangular matrix are its pivots.

w. The singular value decomposition of a diagonalizable matrix ($A = S\Lambda S^{-1}$) is $S\Lambda S^{-1}$.

x. The determinant of a permutation matrix is given by $(-1)^{\text{number of row exchanges}}$.

y. The eigenvalues of A^2 are the squares of the eigenvalues of A.

z. The eigenvalues of $A + B$ are the sums of the eigenvalues of A and B.

Also make sure you can do the questions from the practice midterm, I will ask questions from the first half of the course.